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The Monto-Carlo procedure of some estimation methods for Exponential-Weibull distribution

Israa A. Ahmed, Iden H. Al Kanani

Department of Mathematics, College of Science for Women, University of Baghdad, Baghdad, Iraq

Abstract

In this paper, deicing and derivative two parameters of Exponential Weibull (EW) distribution using three methods of estimate first is maximum likelihood estimation (MLE) the second is ordinary least squares estimation (OLS) and the third is ranked set sampling estimation (RSSE). After that, the simulation technique (Monto-Carlo method) is used to estimate the parameters and Reliability function for different values of sample size and various values of initial values by employing a MATLAB programmer and then using the Mean square error to find which method is the best.

Mathematics Subject Classification (2020): 62F10, 62N05, 65C05

Key words and phrases: Exponential-Weibull distribution; Monte Carlo simulation; Reliability function

1. Introduction

Estimation is a branch of statistics inference that deals with estimating parameter values of any distribution from measurable based on measured empirical data that has a random component. The primary goal of estimation in statistics is to be able to measure the behavior of data within a population. Two types of estimation include Point Estimation and Interval Estimation. In (1993) Mudholkar and Srivastava introduced the exponentiated Weibull family [1]. In (1999) R. Jiang & D.N.P. Murthy introduced The plotting the diagrams of parameters of Weibull distribution then discussed these plotting diagrams [2]. In (2006) Manisha Pal & M. Masoom Ali studied the mathematical and statistical properties for the family of distribution called as Expontiated Weibull distribution [3]. In (2011) Iden &

Email addresses: israa.abbas2303m@csw.uobaghdad.edu.iq (Israa A. Ahmed); Idanha_math@csw.uobaghdad.edu.iq (Iden H. Al Kanani)

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AJ Shaima studied the three parameters Weibull distribution then derived the estimation method for survival function [4]. In (2014) Iden and Muna studied three estimation methods of three parameters for Exponential-Weibull distribution and find the statistical formula for the estimator [5]. In (2021) Aseel and Iden introduced a few properties for Exponentiated Weibull distribution then estimation the parameters by using four estimation method [6]. In [2023] Rehab and Iden introduced some methods of estimating parameters for generalized exponential Rayleigh distribution [7]. In [2023] Rehab and Iden studied the statistical properties of modern generalized exponential Rayleigh distribution [8]. In (2024) Ghasaq and Iden studied the statistical properties of the new Exponential-Weibull distribution [9]. In [2024] Zainab and Rehab studied the maximum likelihood estimation method and used it to estimate two parameters of exponential Rayleigh distribution[10]. In [2024] Rehab and Iden applied three estimation methods to estimate three parameters for exponential Rayleigh distribution [11].

The purpose of this study is to derive the estimation of parameters for Exponential Weibull (EW) distribution by utilizing three estimation methods (maximum likelihood, ordinary least squares, and rank set sampling). Further more employed the simulation technique (Monto-Carlo procedure) to find which estimation method is prefer and addition the Reliability function which most prefer. The second section of this study is applying the statistical propertied for exponential Weibull (EW) distribution. The third section covered estimation methods. In fourth section covered Monte-carol procedure. In Sections Five covered the numerical results and the conclusion in Section Six.

2. The statistical properties for Exponential_Weibull distribution

The main statistical properties for Exponential _Weibull distribution are discussed in this section.

This is probability function for the Exponential Weibull distribution:

$$f_{Ew}(x,\theta,\beta)=\begin{cases}\left(\theta+\beta x^{\beta-1}\right)e^{-\left(\theta x+x^{\beta}\right)} & x>0\\ 0\,o/w\end{cases} \tag{1}$$

$$\Omega=\left\{(\theta,\beta);\theta>0,\beta>0\right\}$$

The cumulative distribution function is:

$$F_{EW}=1-e^{-\left(\theta x+x^{\beta}\right)}\quad \text{x>0} \tag{2}$$

The mean of Exponential Weibull distribution is:

$$E(x)=\sum_{n=0}^{\infty}\frac{(-\theta)^n}{n!}\left[\frac{\theta}{\beta}\Gamma\left(\frac{n+2}{\beta}\right)+\Gamma\left(\frac{n+1}{\beta}\right)+1\right] \tag{3}$$

The variance of Exponential-Weibull distribution is:

$$\operatorname{var}(x)=\sum_{n=0}^{\infty}\frac{(-\theta)^n}{n!}\left[\begin{array}{l}\frac{\theta}{\beta}\left(\Gamma\left(\frac{3+n}{\beta}\right)-\sum_{n=0}^{\infty}\frac{(-\theta)^n}{n!}\Gamma\left(\frac{2+n}{\beta}\right)\left[\frac{\theta}{\beta}\left(\Gamma\left(\frac{2+n}{\beta}\right)-2\Gamma\left(\frac{1+n}{\beta}+1\right)\right]\right]\right)\\ +\Gamma\left(\frac{2+n}{\beta}+1\right)-\sum_{n=0}^{\infty}\left(\frac{(-\theta)^n}{n!}\left(\Gamma\left(\frac{1+n}{\beta}+1\right)\right)^2\right.\end{array}\right] \tag{4}$$

The moment generating function (M.g.f) is defined by:

$$M_x(t)=E\left(e^{tx}\right)=\sum_{r=0}^{\infty}\frac{(t-\theta)^r}{r!}\left[\frac{\theta}{\beta}\Gamma\left(\frac{r+1}{\beta}\right)+\Gamma\left(\frac{r}{\beta}+1\right)\right] \tag{5}$$

The Reliability and hazard functions respectively are:

$$R_{EW}\left(x,\theta,\beta\right)=1-F_{EW}=e^{-\left(\theta x+x^{\beta}\right)} \tag{6}$$

$$h\left(x,\theta,\beta\right)=\frac{f_{Ew}\left(x,\theta,\beta\right)}{R_{EW}\left(x,\theta,\beta\right)}=\left(\theta+\beta x^{\beta-1}\right) \tag{7}$$

3. Estimation methods

Three estimation methods were applied in this section (maximum likelihood(MLEM), ordinary least squares(OLS) and rank set sampling (RSSEM)) to estimate parameters for exponential-Weibull (EW) distribution.

3.1 Maximum Likelihood Estimation method (MLEM)

The maximum likelihood estimation method (MLE) is popular and effective for estimating parameters in any given distribution. The idea of MLE is maximize the likelihood function.

$$L\left(\theta,\beta;x_{1,}\,x_{2.......,}x_{n}\right)=\prod_{i=1}^{n}f_{EW}\left(x_{i},\theta,\beta\right)$$

$$L=\prod_{i=1}^{n}\left(\theta+\beta x_{i}^{\beta-1}\right)e^{-\sum_{i=1}^{n}\left(\theta x_{i}+x_{i}^{\beta}\right)}$$

Taking log- likelihood function :

$$\ln L=\sum_{i=1}^{n}Ln\left(\theta+\beta x_{i}^{\beta-1}\right)-\theta\sum_{i=1}^{n}x_{i}-\sum_{i=1}^{n}x_{i}^{\beta}$$

Then derivative the log- likelihood function as follows:

$$\frac{d\ \ln L}{d\theta}=\sum_{i=1}^{n}\frac{1}{\left(\theta+\beta x_{i}^{\beta-1}\right)}-\sum_{i=1}^{n}x_{i=0}$$

$$\frac{d\ \ln L}{d\beta}=\sum_{i=1}^{n}\frac{[\beta(x_{i}^{\beta-1}\ln x_{i})+x_{i}^{\beta-1}.1]}{\left(\theta+\beta x_{i}^{\beta-1}\right)}-\sum_{i=1}^{n}x_{i}^{\beta}\ln x_{i=0}$$

$$f\left(\theta\right)=\sum_{i=1}^{n}\frac{1}{\left(\theta+\beta x_{i}^{\beta-1}\right)}-\sum_{i=1}^{n}x_{i} \tag{8}$$

$$g\left(\beta\right)=\sum_{i=1}^{n}\frac{[\beta(x_{i}^{\beta-1}\ln x_{i})+x_{i}^{\beta-1}.1]}{\left(\theta+\beta x_{i}^{\beta-1}\right)}-\sum_{i=1}^{n}x_{i}^{\beta}\ln x_{i} \tag{9}$$

Equations (7) and (8) nonlinear equation so we use Newton-Raphson method is as follows:

$$\begin{bmatrix}\theta_{k+1}\\ \beta_{k+1}\end{bmatrix}=\begin{bmatrix}\theta_{k}\\ \beta_{k}\end{bmatrix}-J^{-1}\begin{bmatrix}f\left(\theta\right)\\ g\left(\beta\right)\end{bmatrix}$$

$$J=\begin{bmatrix}\dfrac{df(\theta)}{d\theta} & \dfrac{df(\theta)}{d\beta}\\ \dfrac{dg(\beta)}{d\theta} & \dfrac{dg(\beta)}{d\beta}\end{bmatrix}$$

$$\frac{df(\theta)}{d\theta}=\sum_{i=1}^{n}\frac{-1}{\left(\theta+\beta x_i^{\beta-1}\right)^2}=-\sum_{i=1}^{n}\frac{1}{\left(\theta+\beta x_i^{\beta-1}\right)^2}$$

$$\frac{df(\theta)}{d\beta}=\sum_{i=1}^{n}\frac{-\left(\beta x_i^{\beta-1}\ln x_i+x_i^{\beta-1}\right)}{\left(\theta+\beta x_i^{\beta-1}\right)^2}=-\sum_{i=1}^{n}\frac{\left(\beta x_i^{\beta-1}\ln x_i+x_i^{\beta-1}\right)}{\left(\theta+\beta x_i^{\beta-1}\right)^2}$$

$$\frac{dg(\beta)}{d\theta}=\sum_{i=1}^{n}\frac{-\left(\beta x_i^{\beta-1}\ln x_i+x_i^{\beta-1}\right)}{\left(\theta+\beta x_i^{\beta-1}\right)^2}=-\sum_{i=1}^{n}\frac{\left(\beta x_i^{\beta-1}\ln x_i+x_i^{\beta-1}\right)}{\left(\theta+\beta x_i^{\beta-1}\right)^2}$$

$$\frac{dg(\beta)}{d\beta}=\frac{\left(\theta+\beta x_i^{\beta-1}\right)\left[\beta x_i^{\beta-1}(\ln x_i)^2+2x_i^{\beta-1}\ln x_i\right]-\left(\beta x_i^{\beta-1}\ln x_i+x_i^{\beta-1}\right)^2}{\left(\theta+\beta x_i^{\beta-1}\right)^2}$$

3.2 Ordinary Least Squares Estimation Method (OLSEM)

The second estimation method is know as Ordinary least squares estimation, is widely used to estimate parameters in models with linear or nonlinear variables. The goal of this method is to use a linear approximation to minimize the total of squared differences between the observed sample values and the expected estimate values, as follows:

$$\sum_{i=1}^{n}\int_i^2=\sum_{i=1}^{n}\left(y_i-\hat{y}\ \right)^2$$

Applying this technique will now minimize the difference in squared sum between the estimated cumulative function and the empirical cumulative functions as follows:

$$\sum_{i=1}^{n}\int_i^2=\sum_{i=1}^{n}\left[F(\hat{x}_i)\ -F(x_i)\right]^2$$

The empirical cdf is $F(x_i)=\dfrac{i-0.5}{n}$

$$F_{EW}(x_i)=1-e^{-\left(\theta x_i+x_i^{\beta}\right)}$$

$$\sum_{i=1}^{n}\int_i^2=\sum_{i=1}^{n}\left[1-e^{-\left(\theta x_i+x_i^{\beta}\right)}-\frac{i-0.5}{n}\right]^2$$

$$\sum_{i=1}^{n}\int_i^2=\sum_{i=1}^{n}\left[\frac{n-i+0.5}{n}-e^{-\left(\theta x_i+x_i^{\beta}\right)}\right]^2$$

$$Let \sum_{i=1}^{n} \int_i^2 = S(\theta,\beta)$$

$$S(\theta,\beta) = \sum_{i=1}^{n} \left[\frac{n-i+0.5}{n} - e^{-\left(\theta x_i + x_i^{\beta}\right)} \right]^2$$

$$\frac{ds}{d\theta} = \sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i \, e^{-\left(\theta x_i + x_i^{\beta}\right)} - \sum_{i=1}^{n} x_i \, e^{-2\left(\theta x_i + x_i^{\beta}\right)}$$

$$\frac{ds}{d\beta} = \sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i^{\beta} \, \ln(x_i) e^{-\left(\theta x_i + x_i^{\beta}\right)} - \sum_{i=1}^{n} x_i^{\beta} \, \ln(x_i) e^{-2\left(\theta x_i + x_i^{\beta}\right)}$$

$$f(\theta) = \frac{ds}{d\theta} = \sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i \, e^{-\left(\theta x_i + x_i^{\beta}\right)} - \sum_{i=1}^{n} x_i \, e^{-2\left(\theta x_i + x_i^{\beta}\right)}$$

$$g(\beta) = \frac{ds}{d\beta} = \sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i^{\beta} (\ln x_i) e^{-\left(\theta x_i + x_i^{\beta}\right)} - \sum_{i=1}^{n} x_i^{\beta} (\ln x_i) e^{-2\left(\theta x_i + x_i^{\beta}\right)}$$

The formula of Newton-Raphson method is as follows:

$$\begin{bmatrix} \theta_{k+1} \\ \beta_{k+1} \end{bmatrix} = \begin{bmatrix} \theta_k \\ \beta_k \end{bmatrix} - J^{-1} \begin{bmatrix} f(\theta) \\ g(\beta) \end{bmatrix}$$

$$J = \begin{bmatrix} \frac{df(\theta)}{d\theta} & \frac{df(\theta)}{d\beta} \\ \frac{dg(\beta)}{d\theta} & \frac{dg(\beta)}{d\beta} \end{bmatrix}$$

$$\frac{df(\theta)}{d\theta} = -\sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i^2 \, e^{-\left(\theta x_i + x_i^{\beta}\right)} + \sum_{i=1}^{n} x_i^2 \, e^{-2\left(\theta x_i + x_i^{\beta}\right)}$$

$$\frac{df(\theta)}{d\beta} = -\sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i^{\beta+1} (\ln x_i) e^{-\left(\theta x_i + x_i^{\beta}\right)} + 2\sum_{i=1}^{n} x_i^{\beta+1} (\ln x_i) e^{-2\left(\theta x_i + x_i^{\beta}\right)}$$

$$\frac{dg(\beta)}{d\theta} = -\sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i^{\beta+1} (\ln x_i) e^{-\left(\theta x_i + x_i^{\beta}\right)} + 2\sum_{i=1}^{n} x_i^{\beta+1} (\ln x_i) e^{-2\left(\theta x_i + x_i^{\beta}\right)}$$

$$\frac{dg(\beta)}{d\beta} = \sum_{i=1}^{n} \left(\frac{n-i+0.5}{n} \left[-x_i^{2\beta} (\ln x_i)^2 \, e^{-\left(\theta x_i + x_i^{\beta}\right)} + x_i^{\beta} (\ln x_i)^2 \, e^{-\left(\theta x_i + x_i^{\beta}\right)} \right] \right)$$
$$- \sum_{i=1}^{n} \left[-2x_i^{2\beta} (\ln x_i)^2 \, e^{-2\left(\theta x_i + x_i^{\beta}\right)} + x_i^{\beta} (\ln x_i)^2 \, e^{-2\left(\theta x_i + x_i^{\beta}\right)} \right]$$

3.3 Rank Set Sampling Estimation method (RSSEM)

The third estimation technique discussed here is called the Ranking Set Sampling Estimation method and is symbolled by (RSSEM). The goal of this method is using order statistic to estimate the parameters of exponential-Weibull distribution, after that utilizing maximum likelihood estimation to the order statistic to maximize the parameters of order statistic as follows:

$$F_{EW}(y_i) = 1 - e^{-\left(\theta y_i + y_i^{\beta}\right)}$$

$$g(y_i) = \frac{n!}{(i-1)!(n-i)!}\left[F_{EW}(y_i)\right]^{i-1}\left[1 - F_{EW}(y_i)\right]^{n-i} f_{EW}(y_i)$$

$$g(y_i) = \frac{n!}{(i-1)!(n-i)!}\left[1 - e^{-\left(\theta y_i + y_i^{\beta}\right)}\right]^{i-1}\left[e^{-\left(\theta y_i + y_i^{\beta}\right)}\right]^{n-i}\left(\theta + \beta y_i^{\beta-1}\right)e^{-\left(\theta y + y_i^{\beta}\right)}$$

$$\text{Let } k = \frac{n!}{(i-1)!(n-i)!}$$

$$g(y_i) = k\left(\theta + \beta y_i^{\beta-1}\right)\left[1 - e^{-\left(\theta y_i + y_i^{\beta}\right)}\right]^{i-1}\left[e^{-\left(\theta y_i + y_i^{\beta}\right)}\right]^{n-i+1}$$

likelihood function of sample y_i is

$$L(\theta, \beta, Y_2, \ldots\ldots, Y_n) = \prod_{i=1}^{n} g(y_i, \theta, \beta)$$

$$L = K^n \prod_{i=1}^{n}\left(\theta + \beta y_i^{\beta-1}\right)\prod_{i=1}^{n}\left[1 - e^{-\left(\theta y_i + y_i^{\beta}\right)}\right]^{i-1} e^{-\sum_{i=1}^{n}(n-i+1)(\theta y_i} + y_i^{\beta})$$

Taking the natural log for likelihood function we get

$$\ln L = n\ \ln k + \sum_{i=1}^{n}\ln\left(\theta + \beta y_i^{\beta-1}\right) + \sum_{i=1}^{n}(i-1)\ \ln\left(1 - e^{-\left(\theta y_i + y_i^{\beta}\right)}\right) - \sum_{i=1}^{n}(n-i+1)(\theta y_i + y_i^{\beta})$$

$$\frac{d\ln L}{d\theta} = \sum_{i=1}^{n}\frac{1}{(\theta + \beta y_i^{\beta-1}} + \sum_{i=1}^{n}\frac{(i-1)\, y_i e^{-\left(\theta y_i + y_i^{\beta}\right)}}{\left(1 - e^{-(\theta y_i + y_i^{\beta}}\right)} - \sum_{i=1}^{n}(n-i+1)\, y_i = 0$$

$$\frac{d\ln L}{d\beta} = \sum_{i=1}^{n}\frac{\left(\beta y_i^{\beta-1}\ln y_i + y_i^{\beta-1}\right)}{\left(\theta + \beta y_i^{\beta-1}\right)} + \sum_{i=1}^{n}\frac{(i-1)\, y_i^{\beta}\ln y_i e^{-\left(\theta y_i + y_i^{\beta}\right)}}{\left(1 - e^{-\left(\theta y_i + y_i^{\beta}\right)}\right)} - \sum_{i=1}^{n}(n-i+1)\, y_i^{\beta}\ln y_i = 0$$

$$f(\theta) = \frac{d\ln L}{d\theta} = \sum_{i=1}^{n}\frac{1}{\left(\theta + \beta y_i^{\beta-1}\right)} + \sum_{i=1}^{n}\frac{(i-1)\, y_i e^{-\left(\theta y_i + y_i^{\beta}\right)}}{\left(1 - e^{-(\theta y_i + y_i^{\beta}}\right)} - \sum_{i=1}^{n}(n-i+1)\, y_i$$

$$g(\beta)=\frac{d\ln L}{d\beta}=\sum_{i=1}^{n}\frac{\left(\beta y_i^{\beta-1}\ln y_i+y_i^{\beta-1}\right)}{\left(\theta+\beta y_i^{\beta-1}\right)}+\sum_{i=1}^{n}\frac{(i-1)\,y_i^{\beta}\ln y_i e^{-\left(\theta y_i+y_i^{\beta}\right)}}{\left(1-e^{-\left(\theta y_i+y_i^{\beta}\right)}\right)}-\sum_{i=1}^{n}(n-i+1)\,y_i^{\beta}\ln y_i$$

The formula of Newton-Raphson method is as follows:

$$\begin{bmatrix}\theta_{k+1}\\ \beta_{k+1}\end{bmatrix}=\begin{bmatrix}\theta_k\\ \beta_k\end{bmatrix}-J^{-1}\begin{bmatrix}f(\theta)\\ g(\beta)\end{bmatrix}$$

$$J=\begin{bmatrix}\dfrac{df(\theta)}{d\theta} & \dfrac{df(\theta)}{d\beta}\\[2mm] \dfrac{dg(\beta)}{d\theta} & \dfrac{dg(\beta)}{d\beta}\end{bmatrix}$$

$$\frac{df(\theta)}{d\theta}=\sum_{i=1}^{n}\frac{-1}{\left(\theta+\beta y_i^{\beta-1}\right)^2}-\frac{\sum_{i=1}^{n}(i-1)\,y_i^{2}\,e^{-\left(\theta y_i+y_i^{\beta}\right)}}{\left(1-e^{-(\theta y_i+y_i^{\beta}}\right)^2}$$

$$\frac{df(\theta)}{d\beta}=-\sum_{i=1}^{n}\frac{\left(\beta y_i^{\beta-1}\ln y_i+y_i^{\beta-1}\right)}{\left(\theta+\beta y_i^{\beta-1}\right)^2}-\sum_{i=1}^{n}\frac{(i-1)\,y_i^{\beta+1}\ln y_i\,e^{-\left(\theta y_i+y_i^{\beta}\right)}}{\left(1-e^{-(\theta y_i+y_i^{\beta}}\right)^2}$$

$$\frac{dg(\beta)}{d\theta}=-\sum_{i=1}^{n}\frac{\left(\beta y_i^{\beta-1}\ln y_i+y_i^{\beta-1}\right)}{\left(\theta+\beta y_i^{\beta-1}\right)^2}-\sum_{i=1}^{n}\frac{(i-1)\,y_i^{\beta+1}\ln y_i\,e^{-\left(\theta y_i+y_i^{\beta}\right)}}{\left(1-e^{-\left(\theta y_i+y_i^{\beta}\right)}\right)^2}$$

$$\begin{aligned}\frac{dg(\beta)}{d\beta}=&\sum_{i=1}^{n}\frac{\left(\theta+\beta y_i^{\beta-1}\right)\left[\;(\beta\, y_i^{\beta-1}(\ln y_i)^2+2\,y_i^{\beta-1}\;\ln y_i\right]-\left(\beta y_i^{\beta-1}\ln y_i+y_i^{\beta-1}\right)^2}{\left(\theta+\beta y_i^{\beta-1}\right)^2}\\ &+\sum_{i=1}^{n}\frac{(i-1)\left[-2y_i^{2\beta}(\ln y_i)^2\,e^{-\left(\theta y_i+y_i^{\beta}\right)}+y_i^{2\beta}(\ln y_i)^2\,e^{-2\left(\theta y_i+y_i^{\beta}\right)}+y_i^{\beta}(\ln y_i)^2\,e^{-\left(\theta y_i+y_i^{\beta}\right)}\right]}{\left(1-e^{-\left(\theta y_i+y_i^{\beta}\right)}\right)^2}\\ &-\sum_{i=1}^{n}(n-i+1)\,y_i^{\beta}(\ln y_i)^2\end{aligned}$$

4. Simulation technique

Aset of computing algorithms used in mathematical statistics called Monte Carlo procedure involves repeating an experiment with random initial values.

$$F_{EW} = 1 - e^{-\left(\theta x + x^{\beta}\right)}$$

$$\text{Then } u = 1 - e^{-\left(\theta x + x^{\beta}\right)}$$

$$\left(\theta x + x^{\beta}\right) = \ln \frac{1}{1-u}$$

$$if\ \beta = 1 \text{ then } x = \frac{1}{1+\theta} \ln \frac{1}{1-u}$$

Using the above equation to generating different size of the samples n= 20,30,50 with many initial values of parameters ($\theta = 0.5, 1, 1.5$ and $\beta = 0.25, 0.5, 0.75$). Therefore, the mean squares error values is calculated to compare these methods.

$$MSE\left(\theta^{\wedge}\right) = \sum_{i=1}^{L} \frac{\left(\theta^{\wedge} - \theta\right)^2}{L}, MSE\left(\beta^{\wedge}\right) = \sum_{i=1}^{L} \frac{\left(\beta^{\wedge} - \beta\right)^2}{L}, MSE\left(R(x)^{\wedge}\right) = \sum_{i=1}^{L} \frac{\left(R^{\wedge}(x) - R(x)\right)^2}{L},$$

where n is sample size and L is number repeating of experiment (L=1000).

5. Numerical result

In this section we use Monte Carlo procedure to calculate mean square error for all estimation methods using in this paper.

The Numerical methods algorithm:

1. Start
2. Define initial parameters (n, θ, β)
3. Iteration start with(L=1)
4. Generate random variable(xi) (xi Exponential Weibull distribution) with (θ, β) and (n) sample size
5. Estimate $(\theta, \beta)$by(MLE, RSSE and OLS)
6. Repeat steps (4-5) until iteration (L=1000)
7. The estimators of the (θ, β)parameters will be the average of (1000) estimators for each parameter.
8. Find the mean square error for each parameter
9. End

Table 1: Represent mean square error for $\hat{\theta}$, when $\theta = 0.5$ and $\beta = 0.25, 0.5, 0.75$.

β	n	MLEM	OLS	RSSEM	MIN	Best method
0.25	20	6.92E-07	2.77E-05	1.61E-06	6.92E-06	MLEM
	30	2.90E-09	1.69E-09	1.51E-09	1.51E-09	RSSE
	50	3.74E-10	2.24E-11	1.04E-11	1.04E-11	RSSE
0.5	20	3.11E-02	1.61E-05	1.03E-05	1.03E-05	RSSE
	30	2.005721	1.65E-09	5.35E-10	5.35E-10	RSSE
	50	4.38E-03	1.32E-11	1.44E-11	1.32E-11	OLS
0.75	20	0.330153	1.25E-05	2.00E-05	1.25E-05	OLS
	30	2.55E-02	8.69E-10	2.46E-09	8.69E-10	OLS
	50	1.40E-02	2.28E-11	2.87E-11	2.28E-11	OLS

Table 2: Represent mean square error for $\hat{\beta}$, when $\theta = 0.5$ and $\beta = 0.25, 0.5, 0.75$.

β	n	MLEM	OLS	RSSEM	MIN	Best method
0.25	20	1.25E-03	9.45E-06	1.46E-05	9.45E-06	OLS
	30	1.43E-03	1.03E-09	1.07E-09	1.03E-09	OLS
	50	1.24E-03	1.45E-11	1.19E-11	1.19E-11	RSSE
0.5	20	2.06E-02	1.61E-05	2.84E-05	1.61E-05	OLS
	30	8.39E-03	1.89E-09	1.20E-10	1.20E-10	RSSE
	50	3.36E-03	1.75E-11	1.89E-11	1.75E-11	OLS
0.75	20	5.52E-02	1.39E-05	1.74E-05	1.39E-05	OLS
	30	2.96E-02	2.23E-09	1.35E-09	1.35E-09	RSSE
	50	8.65E-03	1.25E-11	1.65E-11	1.25E-11	OLS

Noting table (1),(2) that the percentage of $\hat{\theta}$ and $\hat{\beta}$ of all the estimation method are as follows:

percentage of $\hat{\theta}$ for MLE $= \frac{1}{9} = 0.111$ percentage of $\hat{\beta}$ for MLE $= 0$

percentage of $\hat{\theta}$ for OLS $= \frac{4}{9} = 0.444$ percentage of $\hat{\beta}$ for OLS $= \frac{6}{9} = 0.666$

percentage of $\hat{\theta}$ for RSSEM $= \frac{4}{9} = 0.444$ percentage of $\hat{\beta}$ for RSSEM $= \frac{3}{9} = 0.333$

Table 3: Represent mean square error for $\hat{\theta}$, when $\theta = 1$ and $\beta = 0.25, 0.5, 0.75$.

β	n	MLEM	OLS	RSSEM	MIN	Best method
0.25	20	2.26E-05	1.83E-05	1.06E-05	1.06E-05	RSSE
	30	2.31E-08	2.63E-09	2.19E-09	2.19E-09	RSSE
	50	5.82E-12	1.37E-11	1.34E-11	5.82E-12	MLEM
0.5	20	3.56E-03	1.80E-05	1.30E-05	1.30E-05	RSSE
	30	2.41E-06	1.30E-09	2.25E-09	1.30E-09	OLS
	50	2.57E-04	1.96E-11	9.04E-12	9.04E-12	RSSE
0.75	20	2.02E-03	1.33E-05	2.19E-05	1.33E-05	OLS
	30	7.03E-04	1.75E-09	1.92E-09	1.75E-09	OLS
	50	6.29E-04	1.99E-11	2.12E-11	1.99E-11	OLS

Table 4: Represent mean square error for $\hat{\beta}$, when $\theta = 1$ and $\beta = 0.25, 0.5, 0.75$.

β	n	MLEM	OLS	RSSEM	MIN	Best method
0.25	20	1.94E-03	1.74E-05	2.61E-05	1.74E-05	OLS
	30	1.79E-03	1.98E-09	1.13E-09	1.13E-09	RSSE
	50	7.17E-04	2.22E-11	2.40E-11	2.22E-11	OLS
0.5	20	1.91E-02	1.73E-05	2.40E-05	1.73E-05	OLS
	30	6.86E-03	1.78E-09	1.11E-09	1.11E-09	RSSE
	50	3.60E-03	1.90E-11	1.36E-11	1.36E-11	RSSE
0.75	20	0.042643	1.18E-05	1.08E-05	1.08E-05	RSSE
	30	1.18E-02	1.50E-09	1.95E-09	1.50E-09	OLS
	50	1.14E-02	2.41E-11	1.52E-11	1.52E-11	RSSE

Noting table (3),(4) that the percentage of $\hat{\theta}$ and $\hat{\beta}$ of all the estimation method are as follows:

Percentage of $\theta^{\wedge}$ $for\, MLE = \frac{1}{9} = 0.111$ percentage of $\beta^{\wedge}$ $for\, MLE = 0$

Percentage of $\theta^{\wedge}$ for OLS $= \frac{4}{9} = 0.444$ percentage of $\beta^{\wedge}$ for OLS $= \frac{4}{9} = 0.444$

Percentage of $\theta^{\wedge}$ for RSSEM $= \frac{4}{9} = 0.444$ percentage of $\beta^{\wedge}$ for RSSEM $= \frac{5}{9} = 0.555$

Table 5: Represent mean square error for $\hat{\theta}$, when $\theta = 1.5$ and $\beta = 0.25, 0.5, 0.75$.

β	n	MLEM	OLS	RSSEM	MIN	Best method
0.25	20	13.7623	1.28E-05	2.35E-05	1.28E-05	OLS
	30	1.96E-03	1.29E-09	1.20E-09	1.20E-09	RSSE
	50	8.36E-10	1.93E-11	6.67E-12	6.67E-12	RSSE
0.5	20	0.751664	1.63E-05	1.68E-05	1.63E-05	OLS
	30	0.382327	1.04E-10	1.51E-10	1.04E-10	OLS
	50	1.331821	2.23E-11	2.41E-11	2.23E-11	OLS
0.75	20	0.914807	1.71E-05	1.05E-05	1.05E-05	RSSE
	30	0.907067	1.40E-09	1.42E-09	1.40E-09	OLS
	50	0.116091	2.53E-11	2.34E-12	2.34E-12	RSSE

Table 6: Represent mean square error for $\hat{\beta}$ when $\theta = 1.5$ and $\beta = 0.25, 0.5, 0.75$.

β	n	MLEM	OLS	RSSEM	MIN	Best method
0.25	20	3.48E-03	1.54E-06	1.87E-06	1.54E-05	OLS
	30	3.82E-03	6.95E-10	2.79E-09	6.95E-10	OLS
	50	1.27E-03	1.68E-11	1.92E-11	1.68E-11	OLS
0.5	20	5.47E-03	2.38E-05	1.13E-06	1.13E-06	RSSE
	30	7.65E-03	2.38E-09	1.02E-09	1.02E-09	RSSE
	50	6.58E-03	3.19E-11	2.16E-11	2.16E-11	RSSE
0.75	20	1.55E-02	1.17E-05	1.94E-05	1.17E-05	OLS
	30	1.12E-02	1.40E-09	1.43E-09	1.40E-09	OLS
	50	1.49E-02	2.45E-11	7.20E-12	7.20E-12	RSSE

Noting table (5),(6) that the percentage of $\hat{\theta}$ and $\hat{\beta}$ of all the estimation method are as follows:

Percentage of $\hat{\theta}$ for $MLE = 0$ percentage of $\hat{\beta}$ for $MLE = 0$

Percentage of $\hat{\theta}$ for OLS = $\frac{5}{9}$0.555 percentage of $\hat{\beta}$ for OLS = $\frac{5}{9}$=0.555

Percentage of $\hat{\theta}$ for RSSEM = $\frac{4}{9} = 0.444$ percentage of $\hat{\beta}$ for RSSEM = $\frac{4}{9}$=0.444

Table 7: The mean square error for Reliability estimators with ($\vartheta = 0.55$ and $\beta = 0.25, 0.5, 0.75$).

β	n	x_i	*MLEM*	*OLS*	*RSSEM*	MIN	Best method
0.25	20	0.1	6.45E-04	7.03E-06	7.35E-06	7.03E-06	OLS
		0.2	3.19E-04	3.63E-06	3.89E-06	3.63E-06	OLS
		0.3	1.70E-04	2.08E-06	2.31E-06	2.08E-06	OLS
		0.4	9.23E-05	1.31E-06	1.51E-06	1.31E-06	OLS
		0.5	4.90E-05	9.28E-07	1.10E-06	9.28E-07	OLS
	30	0.1	6.73E-04	8.96E-10	5.55E-10	5.55E-10	RSSE
		0.2	3.48E-04	4.37E-10	3.08E-10	3.08E-10	RSSE
		0.3	1.89E-04	2.31E-10	1.94E-10	1.94E-10	RSSE
		0.4	1.03E-04	1.30E-10	1.36E-10	1.30E-10	OLS
		0.5	5.47E-05	8.17E-11	1.04E-10	8.17E-11	OLS

(*continues*)

Table 7. Continued

β	n	x_i	*MLEM*	*OLS*	*RSSEM*	MIN	Best method
	50	0.1	6.15E-04	9.45E-12	5.74E-12	5.74E-12	RSSE
		0.2	3.09E-04	5.12E-12	2.86E-12	2.86E-12	RSSE
		0.3	1.66E-04	3.07E-12	1.57E-12	1.57E-12	RSSE
		0.4	8.98E-05	1.95E-12	9.29E-13	9.29E-13	RSSE
		0.5	4.74E-05	1.32E-12	6.16E-13	6.16E-13	RSSE
0.5	20	0.1	4.16E-03	3.55E-06	7.55E-06	3.55E-06	OLS
		0.2	3.11E-03	2.07E-06	5.43E-06	2.07E-06	OLS
		0.3	2.29E-03	1.11E-06	3.67E-06	1.11E-06	OLS
		0.4	1.78E-03	6.14E-07	2.46E-06	6.14E-07	OLS
		0.5	1.49E-03	4.19E-07	1.67E-06	4.19E-07	OLS
	30	0.1	1.19E-02	3.01E-10	3.13E-10	3.01E-10	OLS
		0.2	0.018164	2.20E-10	2.23E-10	2.20E-10	OLS
		0.3	2.08E-02	1.57E-10	1.48E-10	1.48E-10	RSSE
		0.4	2.10E-02	1.16E-10	9.84E-11	9.84E-11	RSSE
		0.5	1.98E-02	9.18E-11	6.72E-11	6.72E-11	RSSE
	50	0.1	7.69E-04	5.92E-12	5.26E-12	5.26E-12	RSSE
		0.2	5.32E-04	4.07E-12	3.99E-12	3.99E-12	RSSE
		0.3	3.59E-04	2.61E-12	2.89E-12	2.61E-12	OLS
		0.4	2.57E-04	1.67E-12	2.11E-12	1.67E-12	OLS
		0.5	2.03E-04	1.09E-12	1.59E-12	1.09E-12	OLS
0.75	20	0.1	9.51E-03	1.59E-06	1.38E-06	1.38E-06	RSSE
		0.2	1.25E-02	1.50E-06	1.20E-06	1.20E-06	RSSE
		0.3	1.31E-02	1.12E-06	8.77E-07	8.77E-07	RSSE
		0.4	1.25E-02	7.80E-07	6.80E-07	6.80E-07	RSSE
		0.5	0.01148	5.55E-07	6.19E-07	5.55E-07	OLS
	30	0.1	1.89E-03	2.42E-10	1.82E-10	1.82E-10	RSSE
		0.2	2.16E-03	2.62E-10	2.23E-10	2.23E-10	RSSE
		0.3	1.91E-03	2.31E-10	2.21E-10	2.21E-10	RSSE
		0.4	1.59E-03	1.90E-10	2.06E-10	1.90E-10	OLS
		0.5	1.34E-03	1.57E-10	1.89E-10	1.57E-10	OLS
	50	0.1	1.11E-03	2.61E-12	1.49E-12	1.49E-12	RSSE
		0.2	1.19E-03	2.91E-12	1.49E-12	1.49E-12	RSSE
		0.3	1.08E-03	2.56E-12	1.32E-12	1.32E-12	RSSE
		0.4	9.44E-04	2.07E-12	1.21E-12	1.21E-12	RSSE
		0.5	8.28E-04	1.62E-12	1.19E-12	1.19E-12	RSSE

where x_i is the values of random variable for Exponetial Weibull distribution

Noting table (7) that the percentage of $R(\hat{x})$ of all the estimation method are as follows:-

Percentage of $R(\hat{x})$ for $MLE = 0$

Percentage of $R(\hat{x})$ for OLS = $\frac{20}{45}$=0.444

Percentage of $R(\hat{x})$ for RSSEM = $\frac{25}{45}$=0.555

Table 8: The mean square error for Reliability estimators with ($\vartheta = 15$ and $\beta = 0.25, 0.5, 0.75$).

β	n	x_i	*MLEM*	*OLS*	*RSSEM*	MIN	Best method
0.25	20	0.1	9.60E-04	1.19E-05	1.44E-05	1.19E-05	OLS
	20	0.2	5.37E-04	6.77E-06	8.10E-06	6.77E-06	OLS
	20	0.3	3.14E-04	4.26E-06	5.07E-06	4.26E-06	OLS
	20	0.4	1.83E-04	2.88E-06	3.42E-06	2.88E-06	OLS
	20	0.5	1.03E-04	2.11E-06	2.45E-06	2.11E-06	OLS
	30	0.1	8.92E-04	1.93E-09	6.61E-10	6.61E-10	RSSE
	30	0.2	5.04E-04	1.07E-09	4.20E-10	4.20E-10	RSSE
	30	0.3	2.98E-04	6.49E-10	3.06E-10	3.06E-10	RSSE
	30	0.4	1.77E-04	4.14E-10	2.47E-10	2.47E-10	RSSE
	30	0.5	1.02E-04	2.78E-10	2.18E-10	2.18E-10	RSSE
	50	0.1	3.92E-04	1.68E-11	1.26E-11	1.26E-11	RSSE
	50	0.2	2.12E-04	9.21E-12	6.75E-12	6.75E-12	RSSE
	50	0.3	1.23E-04	5.47E-12	3.91E-12	3.91E-12	RSSE
	50	0.4	7.20E-05	3.43E-12	2.39E-12	2.39E-12	RSSE
	50	0.5	4.11E-05	2.25E-12	1.57E-12	1.57E-12	RSSE
0.5	20	0.1	4.56E-03	3.49E-06	6.55E-06	3.49E-06	OLS
	20	0.2	3.30E-03	2.46E-06	4.86E-06	2.46E-06	OLS
	20	0.3	2.17E-03	1.67E-06	3.41E-06	1.67E-06	OLS
	20	0.4	1.35E-03	1.20E-06	2.41E-06	1.20E-06	OLS
	20	0.5	8.15E-04	9.63E-07	1.79E-06	9.63E-07	OLS
	30	0.1	1.77E-03	3.25E-10	2.96E-10	2.96E-10	RSSE
	30	0.2	1.34E-03	2.10E-10	2.30E-10	2.10E-10	OLS
	30	0.3	9.14E-04	1.28E-10	1.81E-10	1.28E-10	OLS
	30	0.4	5.89E-04	8.39E-11	1.58E-10	8.39E-11	OLS
	30	0.5	3.58E-04	6.78E-11	1.54E-10	6.78E-11	OLS
	50	0.1	1.04E-03	3.03E-12	4.27E-12	3.03E-12	OLS
	50	0.2	7.29E-04	2.73E-12	3.61E-12	2.73E-12	OLS
	50	0.3	4.75E-04	2.38E-12	2.89E-12	2.38E-12	OLS
	50	0.4	2.96E-04	2.12E-12	2.31E-12	2.12E-12	OLS
	50	0.5	1.77E-04	1.96E-12	1.88E-12	1.88E-12	RSSE
0.75	20	0.1	6.47E-03	1.18E-06	1.15E-06	1.15E-06	RSSE
		0.2	5.84E-03	1.46E-06	1.33E-06	1.33E-06	RSSE
		0.3	4.50E-03	1.46E-06	1.35E-06	1.35E-06	RSSE
		0.4	3.19E-03	1.38E-06	1.41E-06	1.38E-06	OLS
		0.5	2.11E-03	1.30E-06	1.51E-06	1.30E-06	OLS
	30	0.1	1.43E-03	3.07E-10	2.19E-10	2.19E-10	RSSE
		0.2	1.40E-03	3.70E-10	2.46E-10	2.46E-10	RSSE
		0.3	1.08E-03	3.53E-10	2.26E-10	2.26E-10	RSSE
		0.4	7.36E-04	3.11E-10	2.02E-10	2.02E-10	RSSE
		0.5	4.58E-04	2.65E-10	1.86E-10	1.86E-10	RSSE
	50	0.1	1.28E-03	1.24E-12	1.65E-12	1.24E-12	OLS
		0.2	1.34E-03	1.37E-12	1.87E-12	1.37E-12	OLS
		0.3	1.09E-03	1.31E-12	1.78E-12	1.31E-12	OLS
		0.4	7.85E-04	1.26E-12	1.69E-12	1.26E-12	OLS
		0.5	5.19E-04	1.28E-12	1.67E-12	1.28E-12	OLS

Noting table (8) that the percentage of $R(\hat{x})$ of all the estimation method are as follows:-

Percentage of $R(\hat{x})$ for $MLE = 0$

Percentage of $R(\hat{x})$ for OLS $= \frac{25}{45}$ =0.555

Percentage of $R(\hat{x})$ for RSSEM $= \frac{20}{45}$ =0.444

Table 9: The mean square error for Reliability estimators with ($\vartheta = 1.5$ and $\beta = 0.25, 0.5, 0.75$).

β	n	x_i	*MLEM*	*OLS*	*RSSEM*	MIN	Best method
0.25	20	0.1	1.30E-02	7.73E-06	7.38E-06	7.38E-06	RSSE
		0.2	1.70E-02	3.07E-06	3.05E-06	3.05E-06	RSSE
		0.3	0.014438	1.30E-06	1.44E-06	1.30E-06	OLS
		0.4	1.08E-02	5.66E-07	7.24E-07	5.66E-07	OLS
		0.5	7.77E-03	2.62E-07	4.29E-07	2.62E-07	OLS
	30	0.1	1.56E-03	5.73E-10	1.16E-09	5.73E-10	OLS
		0.2	6.30E-04	2.26E-10	5.00E-10	2.26E-10	OLS
		0.3	2.78E-04	9.88E-11	2.37E-10	9.88E-11	OLS
		0.4	1.29E-04	4.81E-11	1.18E-10	4.81E-11	OLS
		0.5	6.34E-05	2.78E-11	6.24E-11	2.78E-11	OLS
	50	0.1	4.89E-04	9.30E-12	7.80E-12	7.80E-12	RSSE
		0.2	2.06E-04	4.10E-12	3.28E-12	3.28E-12	RSSE
		0.3	9.17E-05	2.01E-12	1.49E-12	1.49E-12	RSSE
		0.4	4.09E-05	1.07E-12	7.09E-13	7.09E-13	RSSE
		0.5	1.78E-05	6.21E-13	3.51E-13	3.51E-13	RSSE
0.5	20	0.1	2.21E-03	2.16E-06	2.18E-06	2.16E-06	OLS
		0.2	3.92E-03	1.29E-06	1.18E-06	1.18E-06	RSSE
		0.3	4.49E-03	7.25E-07	6.41E-07	6.41E-07	RSSE
		0.4	4.24E-03	4.08E-07	3.82E-07	3.82E-07	RSSE
		0.5	3.61E-03	2.34E-07	2.61E-07	2.34E-07	OLS
	30	0.1	2.94E-03	2.86E-10	2.27E-10	2.27E-10	RSSE
		0.2	3.56E-03	1.49E-10	1.42E-10	1.42E-10	RSSE
		0.3	3.63E-03	7.54E-11	8.70E-11	7.54E-11	OLS
		0.4	3.31E-03	4.20E-11	5.58E-11	4.20E-11	OLS
		0.5	2.82E-03	2.76E-11	3.79E-11	2.76E-11	OLS
	50	0.1	5.70E-03	3.40E-12	5.14E-12	3.40E-12	OLS
		0.2	8.26E-03	1.74E-12	3.32E-12	1.74E-12	OLS
		0.3	8.57E-03	8.29E-13	2.07E-12	8.29E-13	OLS
		0.4	7.63E-03	4.06E-13	1.31E-12	4.06E-13	OLS
		0.5	6.25E-03	2.24E-13	8.53E-13	2.24E-13	OLS
0.75	20	0.1	0.006611	1.37E-06	1.73E-06	1.37E-06	OLS
		0.2	9.88E-03	8.07E-07	1.46E-06	8.07E-07	OLS
		0.3	1.03E-02	3.73E-07	9.83E-07	3.73E-07	OLS
		0.4	9.20E-03	1.88E-07	6.20E-07	1.88E-07	OLS
		0.5	7.50E-03	1.47E-07	3.89E-07	1.47E-07	OLS
	30	0.1	4.79E-03	1.26E-10	1.36E-10	1.26E-10	OLS
		0.2	8.21E-03	9.45E-11	1.22E-10	9.45E-11	OLS
		0.3	8.98E-03	5.78E-11	8.92E-11	5.78E-11	OLS
		0.4	8.12E-03	3.47E-11	6.20E-11	3.47E-11	OLS
		0.5	6.63E-03	2.25E-11	4.30E-11	2.25E-11	OLS
	50	0.1	9.07E-04	1.40E-12	5.76E-13	5.76E-13	RSSE
		0.2	1.31E-03	1.13E-12	5.50E-13	5.50E-13	RSSE
		0.3	1.47E-03	7.41E-13	4.76E-13	4.76E-13	RSSE
		0.4	1.45E-03	4.75E-13	4.16E-13	4.16E-13	RSSE
		0.5	1.33E-03	3.18E-13	3.66E-13	3.18E-13	OLS

Noting table (9) that the percentage of $R(\hat{x})$ of all the estimation method are as follows: -

Percentage of $R(\hat{x})$ for $MLE = 0$

Percentage of $R(\hat{x})$ for OLS $= \frac{29}{45} = 0.644$

Percentage of $R(\hat{x})$ for RSSEM $= \frac{16}{45} = 0.355$

6. Conclusion

From this research getting the following conclusion: -

1. From table (1,3,5) showing that OLS is the best where the percentage of mean square error for $\hat{\theta}$ is $\frac{13}{27}=0.481\%$.
2. From table (2,4,6) showing that OLS is the best where the percentage of mean square error for $\hat{\beta}$ is $\frac{15}{27}=0.555\%$.
3. From table (7,8,9) showing that OLS is the best where the percentage of mean square error for $\hat{R}(x)$ is $\frac{74}{135}=0.548\%$.

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