Results in Nonlinear Analysis 8 (2025) No. 2, 133–147
https://doi.org/10.31838/rna/2025.08.02.012
Available online at www.nonlinear-analysis.com

Results in Nonlinear Analysis
ISSN 2636-7556

Peer Reviewed Scientific Journal

Mathematical modeling and optimization of intelligent systems using a hybrid PSO-GWO algorithm: A $\min_x J(x)$ approach

Jumana J. Al-zamili

College of Computer Science & Information Technology/Al-Qadisiyah University, Iraq

Abstract

In this paper, a comparative analysis of Particle Swarm Optimization (PSO), Grey Wolf Optimization (GWO), and a hybrid PSO-GWO algorithm for the solution of complex optimization problems have been presented. The hybrid algorithm consists of the exploitation strength of GWO and the exploration capabilities of PSO while combining both together to surmount the failure of standalone algorithms like slow convergence in GWO and premature convergence in the PSO. The algorithms were evaluated in terms of convergence speed, robustness, and accuracy, on a series of benchmark functions (Sphere, Rastrigin, Ackley, Rosenbrock, and Griewank). The results of simulations indicate that the notion of a hybrid PSO-GWO algorithm is always better compared to standalone PSO as well as GWO for lower mean square errors (MSE) and quicker convergence to global optimum for different optimization landscapes. Finally, the hybrid approach took advantage of the best in all three aspects to optimize multimodal, non-convex, and deceptive functions with both reliable and robust performance that was superior to that of local minimum avoidance algorithms. This research shows that the hybrid PSO-GWO algorithm is an efficient tool for robust optimization in the real-world systems of dynamic, complex systems. Future work is to extend the algorithm's adaptability to real-world constraints, and dynamic parameter adjustment, and integrate it within domain-specific heuristics to improve the optimization in engineering and automation tasks.

Mathematics Subject Classification (2020): 90C59, 90C26, 68T20

Key words and phrases: Intelligent systems; Optimization algorithms; PSO; GWO; Hybrid PSO-GWO.

Email address: Jjamalhas1985@gmail.com (Jumana J. Al-zamili)

Received April 22, 2025; Accepted May 11, 2025; Online June 28, 2025

1. Introduction

Certainly, one of the most daunting tasks in control systems engineering is the optimization of system performance to achieve desired dynamic behaviour. The importance of this is particularly important in systems with complex dynamics and nonlinear constraints [1]. Control parameters such as damping ratio and natural frequency must be optimized for certain outcomes of interest, namely, minimum overshoot, fast settling time and robust stability under a variety of conditions [2]. As these complex tasks are inadequately treated with conventional approaches, we need an advanced metaheuristic algorithm [3]. Natural and social phenomena inspired metaheuristic algorithms that have gained popularity for optimization tasks, particularly those that come equipped with nonconvexity, high dimensionality, and multiplicity [4]. These two algorithms belong to the category of the prominent known algorithms Particle Swarm Optimization (PSO) and Grey Wolf Optimization (GWO) [5, 6]. Exploration is complemented with exploitation by the use of Particle swarm optimization (PSO), which considers the social behaviour of the particles in the search of the space and repeatedly leads them to their nearest neighbours, thereby making them to converge to optimum solutions [7]. However, unlike Babu's algorithm, GWO exploits patch generation, searching with an adaptive exploration and solution refinement inspired by leaders [8, 9].

Further integration of such optimization techniques and algorithms within the intelligent control systems may be more effectively achieved through artificial intelligence-inspired adaptive decision-making frameworks [10]. The structure of such a system starts with the user activity being recorded and logged, then processed in a Data Preprocessing module [11]. It takes the data from raw format to structured format; this generates training, validation and test datasets or real-time data records. The pattern learnt predictions of the data is sent to a Classifying Module for prediction of outcomes and an RL Module tries to find the best action by using predictive insights and service-related information. As shown in Figure 1, dynamic control and adaptation require an Actuator to perform the determined actions to complete the feedback loop. By this, the synergy between metaheuristic optimization and artificial intelligence is underlined forming this multi-layer architecture for intelligent systems to deal with complex dynamic behaviours and constraints of control engineering and multimedia services [12, 13].

Eventually, PSO and GWO are powerful optimization techniques independently, but when used independently, tend to show the weakest points of the methods in achieving a tradeoff between convergence speed and solution quality [14, 15]. Hybrid optimization approaches that combine strengths among more than one algorithm have recently become an attractive alternative [16]. The convergence of hybrid PSO-GWO will be enhanced with the optimal features of PSO and GWO, in regard

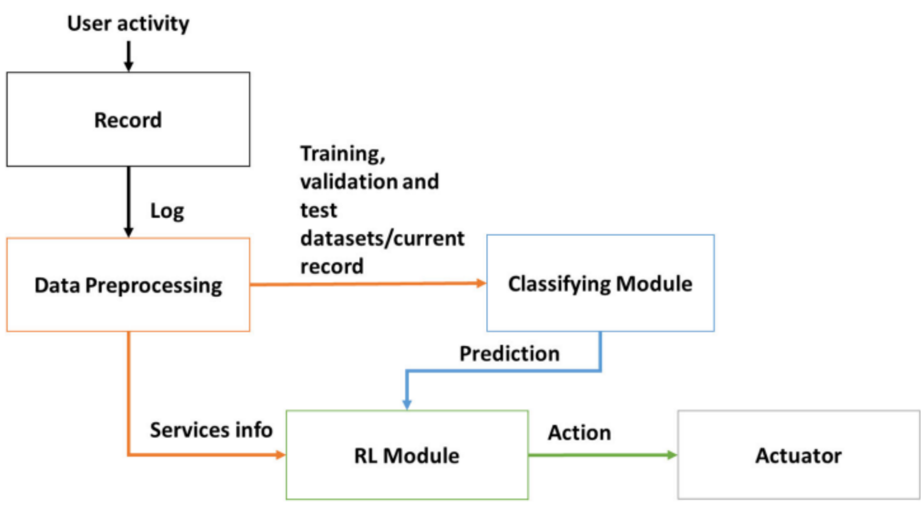

Figure 1: Illustration for the generic diagram of the intelligent systems.

to exploration and exploitation, respectively, to make it more accurate and robust [17]. This research analyzes the performance of PSO, GWO, and Hybrid PSO GWO algorithms in terms of achieving the optimal value of damping ratio (ζ) for the second-on-order control system. The goal is to maintain the system characteristics (overshoot and settling time) near the desired values. The algorithms are tested for convergence, mean square error, and robustness through simulations using MATLAB, many times, and through simulations in these multiple runs. The current study therefore stresses the benefits of hybrid methods in tackling complex problems of optimization and illustrates the feasibility of such methods for the design of advanced control systems.

The remainder of this paper is organized as follows: The second section presents a literature survey of previous work and the theoretical background. The third describes the proposed methodology, which describes the foundation and the approach of the combined PSO and GWO. In the fourth section, the model setup and methodology are explained, including the configuration of the PSO, GWO and Hybrid PSO-GWO algorithms as well as the simulation environment. In the next section, the sixth section, the hybrid optimization process is elaborated upon, showing the combination of PSO and GWO techniques used, and the key steps of their hybridization. In Section 6 we introduce used benchmark functions to test the performance of the algorithms. In the seventh section, the results of a comparative analysis of the performance of PSO, GWO, and the Hybrid PSO-GWO algorithm are carried out. Finally, the eighth section concludes the paper by summarizing the findings, concluding key contributions.

2. Literature Survey

The solving of multi-constrained and global optimisation problems in heterogeneous applications relies on optimisation algorithms. There have been recent studies that report the growing use of hybrid approaches, combining the strength of several algorithms to achieve better and/or better solutions as well as higher performances. The relevant works to this study are reviewed, with the developments in hybrid optimisation techniques and their applications.

In this paper, the authors have investigated the hybrid Genetic Algorithm-PSO approach, called GPSO, which is proposed to solve multi-constrained optimization problems. This research represents the hybrid model by combining the exploration capabilities of Genetic Algorithms with the exploitation efficiency of the PSO, thus yielding significantly better results compared to conventional methods in solving complex optimization tasks [18]. The authors in [19] also make a further contribution in presenting various combinations of swarm-based algorithms, namely, the Crow Search Algorithm (CSA), GWO, Harris Hawks Optimisation (HHO), and Whale Optimization Algorithm (WOA). They were shown that hybrid methods, in this case CSA-GWO, are more effective than their base counterparts in terms of the optimisation efficiency and robustness [20]. Recent research has centred on advancements in GWO methods. The Advanced GWO (AGWO) has performed better in power distribution system planning than standard GWO and PSO. Using this improved algorithm, optimized voltage profiles, lowered power losses and reduced emission costs, offer great benefits to the power plant developers and are therefore a very valuable tool for developing efficient and sustainable power systems [21]. The authors of this study also present a multi-strategy ensemble GWO algorithm which integrates several advanced optimization techniques, furthering its performance. Combined across several strategies, the algorithm was found to converge accurately, speed up computation, and address the problem of local optima. Where problems and changes are most difficult and diverse, for example in robot path planning, precision, adaptability and efficiency are vital with such enhancements. To begin with, it describes the applicability of hybrid GWO models in solving problems in robotics and engineering in general in general, as well as the relisation of this potential in three real-world problems in robotics [22]. An innovative expansion of the framework demonstrated here is the development of a self-learning variant of GWO, the Adaptive Dynamic Self Learning GWO (ASGWO) and self-learning variant. In addition, this superior algorithm learns time-sensitive learning approaches to address the rivalry of the converging speed and local optima issues. The improvements make it possible for ASGWO to deliver high reliabilities in performance with the complex global optimization and other engineering applications [23]. Moreover, there is also existence of using hybrid strategies on less popular algorithms, namely, Golden Jackal Optimization (GJO); among others. It is worth emphasizing that the implementation of hybridization in GJO leads to a substantial enhancement of its performance, which in terms of solving global optimization issues, yields a significantly higher outcome than traditional strategies. These improvements show that the proposed hybridized GJO was capable of outperforming standalone algorithms and obtaining more dependable as well as efficient solutions to numerous optimisations tasks [24]. Last of all, a contrast of the solutions of the GA to the solutions of the PSO to the OPF problem is discussed highlighting the strengths and weakness of each method. The findings of the study show the effectiveness of GA in solving complex global optimization problems and high convergence rate of PSO in finding the local optimum. But at the same time it poses some weaknesses in applications of standalone sizes because it does not address large power systems. For this purpose, the research presents a dual approach to address these challenges based on the strength of Algorithm 1 and Algorithm 2 regarding solution quality, problem-solving, and computational effectiveness of OPF problems [25].

3. Proposed Methodology

In this work, we assess comprehensively three more optimization techniques, namely the PSO technique, the GWO technique, and the hybrid of the PSO and GWO technique that can solve for the zeta that is placed in the second-order control system. Settling time and overshoot largely depend on the damping ratio a key factor that defines the performance of the system. The damping ratio is adjusted to reduce fluctuations in the desired system characteristics using each of the optimization algorithms. This research presents the hybrid PSO-GWO algorithm which integrates the exploration of PSO with the exploitation of GWO to present a robust optimization system which frees the limitations of standalone algorithms. Some of the MATLAB simulation results, which analyse the efficiency of using the hybrid algorithm with reference to dimensions, convergence speed to the optimal solution, the objective function errors as well as the control system robustness of both PSO and GWO are then provided next. In an effort to meet these criteria, the mean performance in multiple runs, standard deviation and MSE are used. Thus, these metrics reveal how well the algorithm of local optimization is in identifying good values of control system parameters given dynamic inputs and a constrained setting. Second, real-world application of the hybrid PSO-GWO algorithm to improve convergence accuracy and robustness are also supported by the various testing situations. The objectives of this research are outlined below, each explained as follows:

- A comparison of the performance of the PSO, GWO and Hybrid of PSO and GWO algorithms in terms of optimizing the damping ratio of a second-order control system having desired overshoot and settling time characteristics.
- Hybrid PSO GWO algorithm was developed and implemented, and integrated into the strengths of the PSO's global exploration and GWO's local refinement to meet better optimization performance.
- Through MATLAB simulations, the robustness and effectiveness of optimization algorithms have been evaluated with metrics such as mean objectives value, standard deviation, and MSE to see how those algorithms perform over multiple runs.
- The applicability of the hybrid PSO-GWO algorithm in improving control system performance in dynamic control is proved by testing its ability to reduce overshoot and diminishing settling time in control systems dynamics.
- Ensuring reliability in practical applications, it provides insights about the statistical behaviour of the algorithms (convergence patterns and stability under constrained optimization environments).

- In order to determine how hybrid optimization techniques might be applied in other engineering areas, such as industrial automation, robotics and power systems, where precise parameter tuning is critical to achieving high performance.

4. Model Setup and Methodology

This section elaborates on the model setup for comparing PSO, GWO, and Hybrid PSO-GWO algorithms in optimizing a control system. This involves explaining the mathematical formulation of the problem, the algorithmic flow, and the pseudo-code for each optimization technique in detail: initializations, system design, optimization, and performance analysis.

4.1 Initialization and System Design

These optimisations start with the definitions of critical parameters in view, desired damping ratio (ζ) overshoot, and settling time of a control system. These defined parameters are driving the optimization process to make it stable and efficient.

A) **Control System Definition**

The system to be optimised is modelled as a second-order transfer function:

$$H(s) = \frac{1}{s^2 + 2\zeta s + 1} \tag{1}$$

Where ζ is defining the damping ratio which is the optimization variable and s is referring to the Laplace transform variable.

The step response characteristics of the system are used to calculate key system performance metrics, including, for example, per cent overshoot and settling time. The objective function is defined as:

$$f(\zeta) = \left|OS_{desired} - OS_{actual}\right| + \left|ST_{desired} - ST_{actual}\right| \tag{2}$$

Where $\mathrm{OS}_{\mathrm{desired}}$ and $\mathrm{ST}_{\mathrm{desired}}$ are the desired overshoot and settling time, $\mathrm{OS}_{\mathrm{actual}}$ and $\mathrm{ST}_{\mathrm{actual}}$ are the system's actual overshoot and settling time for the given ζ.

B) **Simulation and Performance Metrics Setup**

Simulation Setup for this work can be summarised as follows:

- **Iterations and Population Size:** The algorithms run 50 iterations with a population size of 20 particles/wolves.
- **Multiple Runs:** The simulations are run three times to ensure reliability, and results are averaged.
- **Performance Metrics:** Evaluation metrics include the following:
- **Mean Objective Value:** Average objective function value across iterations.
- **Convergence Rate:** Speed at which the algorithm approaches the optimal solution.
- **MSE:** Deviation of the final solution from the ideal target.

Table 1: The scoring system used to evaluate algorithm performance across key metrics.

Algorithm	Convergence Rate	Accuracy	Stability	Score (1-5)
PSO	Moderate	Good	Moderate	3
GWO	Good	Good	Good	4
Hybrid PSO-GWO	Excellent	Excellent	Excellent	5

5. Hybrid Optimization Process

The Hybrid PSO-GWO algorithm utilises PSO and GWO mechanisms to alternate within the optimisation process to explore global space and engage local exploitation.

A) **PSO Phase**

In the PSO phase, the position and the velocity of the particles will be updated based on their private experiences and the global best solution:

$$vi^{k+1} = w \cdot v_i^k + c_1 \cdot r_1 \cdot \left(p_{best} - x_i^k\right) + c_2 \cdot r_2 \cdot \left(g_{best} - x_i^k\right) \tag{3}$$

$$xi^{k+1} = x_i^k + vi^{k+1} \tag{4}$$

Where w is the inertia weight, balancing exploration and exploitation, c_1 and c_2 are acceleration coefficients, r_1 and r_2 are random numbers between 0 and 1, p_{best} is the best personal position of the particle, and g_{best} is the best global position across all particles.

B) **GWO Phase**

In the GWO phase, wolves' positions are updated with respect to the position of three best solutions, alpha (α), beta (β), and delta (δ):

$$D_\alpha = \left|C_1 \cdot \alpha - x_i\right| \tag{5}$$

$$D_\beta = \left|C_2 \cdot \beta - x_i\right| \tag{6}$$

$$D_\delta = \#C_3 \cdot \delta - x_i\# \tag{7}$$

$$x_i = \frac{x_\alpha + x_\beta + x_\delta}{3} \tag{8}$$

Where C_1, C_2, C_3 are referring to the random coefficients guiding exploration.

The hybrid algorithm can be considered as a combination of PSO and GWO phases, combining the global exploration and fast convergence of PSO with the local search and refinement of GWO. This alternation balances exploration and exploitation, enhancing solution accuracy and robustness. The hybrid algorithm divided into a couple of phases:

- **Exploration:** During early iterations, it focuses on searching for a broad solution space in the PSO Phase.
- **Exploitation-GWO Phase:** The solutions are refined during later iterations by focusing on the best candidates. In addition, a combination rule is utilized within the hybrid approach to adaptively switch between PSO and GWO phases based on convergence behaviour.

In order not to prematurely converge and lose diversity, the best solutions are kept for the next generation, well-known as Elitism. Random mutations are performed on a small portion of the population, such as in the following equation:

$$x_{mutated} = x + mutation\_factor \cdot r \cdot \left(\zeta max - \zeta min\right) \tag{9}$$

The pseudo-code for the hybrid PSO-GWO algorithm can be outlined as follows, which provides the sequential steps required for initializing the parameters, running through iterations of PSO and GWO phases, and updating the solutions until the fulfilment of the termination criteria.

Algorithm 1: proposed Pseudo-Code for Hybrid PSO-GWO

```
Initialize positions and velocities of particles/wolves randomly
Evaluate initial fitness of each particle/wolf
While stopping criteria not met:
  If iteration <= threshold
    % PSO Phase
    Update velocities and positions using PSO equations
  else
    % GWO Phase
    Update positions using GWO equations
  end
    Evaluate fitness of updated solutions
    Apply elitism and mutation
    Update best solutions (p_best, g_best, alpha, beta, delta)
    end
    Return optimal damping ratio (zeta_opt)
```

6. Benchmark Functions

Benchmark functions are widely used to test and compare the performance of different optimization algorithms. They can be thought of as controlled environments where algorithms are put to the test in regard to their ability to explore complex search spaces, cope with local minima, and converge toward global optima. A number of benchmark functions that were used in this work are discussed in the following with their mathematical representations, characteristics, and challenges.

6.1 Sphere Function

One of the simplest and most used benchmark functions is the Sphere function. It is convex and unimodal with its global minimum at its origin, that is, at coordinates (0,0). Due to the smooth, parabolic shape of this function, it often serves as a basic test for optimization algorithms.

$$f(x,y) = x^2 + y^2 \tag{10}$$

6.2 Rastrigin Function

Rastrigin is a highly multimodal function with a multitude of local minima due to the cosine terms. Paradoxically, the global minimum can be found at the origin, that is, (0,0). The periodic ripples introduced by the cosine terms make it a pathfinding test case for any optimizer.

$$f(x,y) = 10 \cdot n + \left(x^2 - 10 \cdot \cos(2\pi x)\right) + \left(y^2 - 10 \cdot \cos(2\pi y)\right) \tag{11}$$

6.3 Ackley Function

The Ackley function has a very flat outer region and a central basin associated with many local minima. It is a nice gradient but its steep peaks and valleys make it a difficult function. The origin (0,0) is the (global) minimum.

$$f(x,y) = -20 \cdot \exp\left(-0.2 \cdot \sqrt{0.5 \cdot \left(x^2 + y^2\right)}\right) - \exp\left(0.5 \cdot \left(\cos(2\pi x) + \cos(2\pi y)\right)\right) + e + 20 \tag{12}$$

6.3 Rosenbrock Function (Banana Function)

It features a curvature similar to that of the name, and has a narrow deep valley with its derivative at its global minimum at (1,1) for the Rosenbrock function, also known as the 'Banana function'. The global minimum is easy to find mathematically, but the bad steep sides and narrow valleys are very hard to optimize because the optimization algorithms have a very hard time converging well.

$$f(x,y) = (1-x)^2 + 100 \cdot (y - x^2)^2 \tag{13}$$

6.4 Griewank Function

A multimodal functionality with many local minima mixed over its search space is called the Griewank function. With increasing distance of variables away from 0, the polynomial term is growing and there are oscillations resulting from cosine terms. Since the global minimum is at the origin: (0,0), this is a good test bench for the algorithms that get stuck in their local minimums.

$$f(x,y) = 1 + \frac{4000}{x^2} + \frac{4000}{y^2} - \cos(x) \cdot \cos\left(\frac{2}{y}\right) \tag{14}$$

The global minima of the benchmark function, domain in which they are defined, characteristics and level of difficulty are summarized in Table 1 below.

6.5 Applications for Optimization

These benchmark functions test various capabilities of optimization algorithms, such as:

- **Navigating Multimodal Landscapes:** These functions like Rastrigin and Griewank impose problems on the algorithms to reach out from local minima.
- **Handling Narrow Valleys:** The Rosenbrock function challenges an algorithm to traverse curved, narrow paths.
- **Baseline Testing:** The basic functionality test is provided by the Sphere function, which poses a simple convex problem.
- **Balancing Exploration and Exploitation:** Ackley's main idea is to test whether an algorithm is capable of balancing broad research (exploration) with local biasing (exploitation).

Such benchmark functions present challenging diverse problems for the evaluation of robustness, efficiency, and convergence capabilities of optimization techniques that is applied in our case.

7. Results and Discussion

In this section, we summarise the outcomes and also do a brief analysis of the performance of the three optimisation methods (PSO, GWO, and Hybrid PSO-GWO) in terms of the results and the results from

Table 1: Our updated benchmark functions.

Function	Global Minimum	Domain	Nature	Challenge Level
Sphere	$f(0,0)=0$	[-5.12, 5.12]	Convex, Unimodal	Low
Rastrigin	$f(0,0)=0$	[-5.12, 5.12]	Multimodal	High
Ackley	$f(0,0)=0$	[-5, 5]	Multimodal	Moderate
Rosenbrock	$f(1,1)=0$	[-5, 5]	Non-convex, Unimodal	High
Griewank	$f(0,0)=0$	[-5, 5]	Multimodal	High

simulation and benchmark evaluation, which were run in MATLAB. The results indicate the relative effectiveness of the performance of each algorithm for optimising towards different objectives while measuring convergence sharpness, accuracy, and robustness with respect to different benchmarks. An analysis of each strategy presented reveals the strengths and weaknesses of each approach and specifically shows the Hybrid PSO-GWO algorithm surpasses the others by significantly improving performance in complex optimization problems where accurate results are desired.

7.1 Hybrid PSO-GWO algorithm for Intelligent System

In our study, the Hybrid PSO-GWO implementation of our algorithm is optimized to optimize the system parameters in order to get the optimal performance. The algorithm exploits the powers of GWO along with the exploration powers of PSO. Likewise, this hybrid approach makes the most of the strengths of these algorithms: PSO because it has the potential to escape from local optimum and GWO due to its algorithm slowing down the convergence. To gain both optimization accuracy and computational efficiency, a Hybrid PSO-GWO algorithm with careful choice of key parameters was made. The number of iterations set to 50, decided the length of the optimization process. In each iteration (or step), the particles (candidate solutions) are evaluated and re-evaluated based on performance. By setting the number of iterations to 50, the algorithm converges to a near-optimal solution, while incurring a small computational cost. A population size of 20 particles is chosen as it assures sufficient diversification of the search space without excessively increasing computational cost. Balancing the need for exploration with the computational overhead of a larger population, this number was chosen. The fitness function given was devised to minimize system output variation, by minimizing overshoot and settling time. By lower fitness values, these lower fitness values mean that better performance is indicated, which then tells the algorithm to go to optimal solutions. Specifically, it updated the positions of particles using PSO-inspired velocity updates and GWO-inspired leader updates. With this dual-update mechanism, exploration of the solution space was ensured to be robust, while also quickly converging on the global optimum. The convergence comparison in Figure 2 shows the superiority of the Hybrid PSO-GWO algorithm. Compared with standalone PSO and GWO algorithms, the hybrid approach showed a consistent convergence in a faster and more accurate way. PSO demonstrated slow convergence with large objective values, and GWO performed slower than PSO. The hybrid algorithm showed a steady decrease in objective value, which decreased at lower values with less variability across iterations. This shows its robustness with its reliability in making consistent performance under different runs.

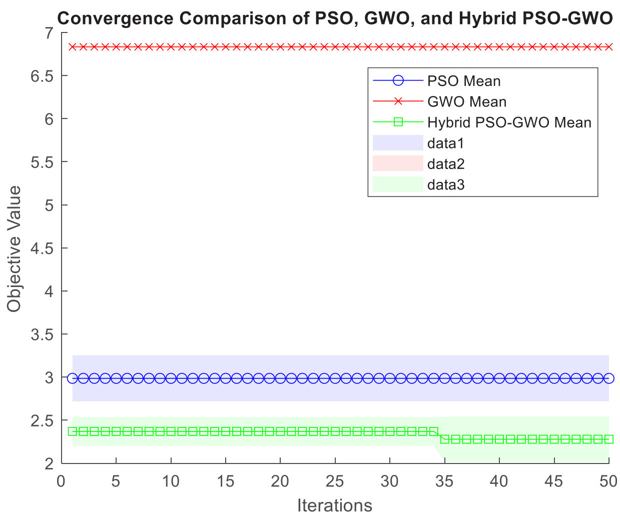

Figure 2: Convergence comparison of the utilised algorithms.

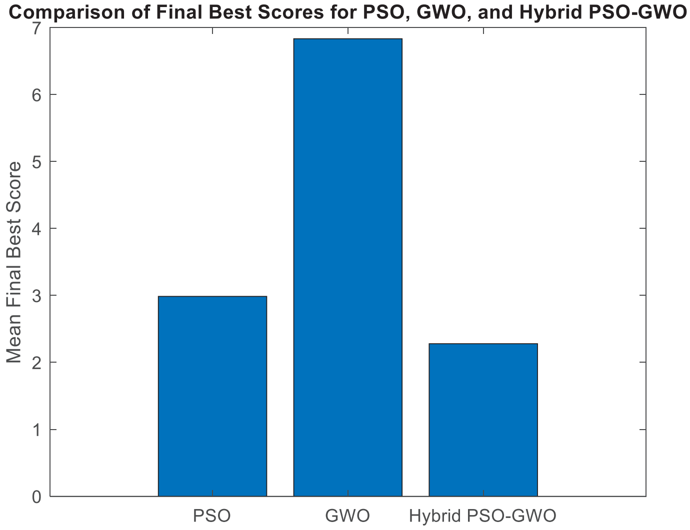

Figure 3: Comparison of final best scores for the utilised algorithms.

Each algorithm's mean final best scores are then compared in Figure 3. The level of convergence and base solutions of the GWO is slower than other methods, which was reflected in the highest value of the score. While slightly lower in score than GWO, the hybrid PSO-GWO case outperformed PSO significantly. Finally, this result shows the hybrid algorithm's effectiveness in balancing exploration and exploitation to produce superior optimization results.

The standard deviation of the final score over multiple runs is shown in Figure 4. The results showed that the PSO and the hybrid PSO GWO algorithm had low variability suggesting relatively consistent performance. On the other hand, GWO exhibited greater variability suggesting poor sensitivity to the initial conditions and instability in escaping local minima. The hybrid algorithm demonstrates its reliability for practical use due to its stability across runs.

The effect of the hybrid algorithm is shown in Figure 5, where the MSE comparison shows. The MSE for hybrid PSO-GWO was lower than the above approaches and the lowest MSE was achieved by the hybrid PSO-GWO approach. MSE showed GWO is having challenges in finding the global optimum. The superior ability of the hybrid algorithm to handle complex optimization problems leads to its significantly lower MSE, and thus its significance in real-world applications.

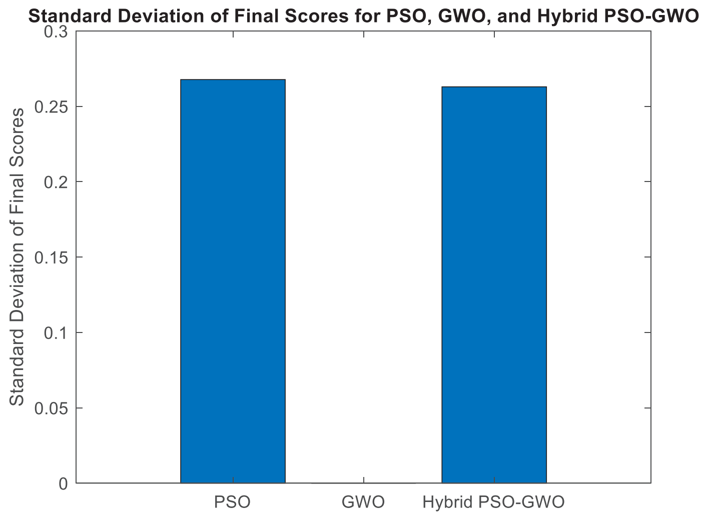

Figure 4: Standard deviation of final scores for the utilised algorithm.

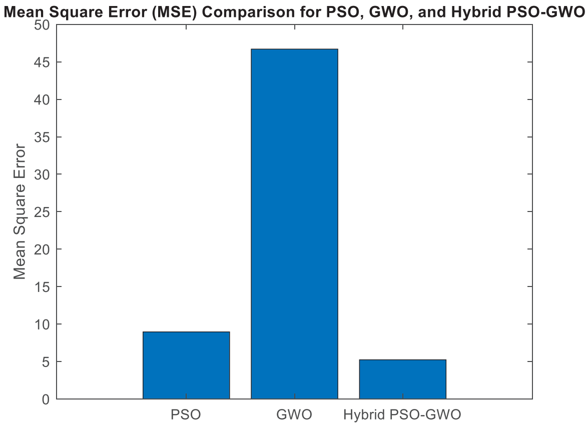

Figure 5: MSE comparison for the utilised algorithms.

7.2 Results of Benchmark Functions

A set of benchmark functions is utilised in this research to evaluate the performance of the Hybrid PSO-GWO. With the hope of representing a variety of optimization challenges, from simple convex to highly multimodal and deceptive landscapes, these benchmark functions were chosen. By testing this algorithm, we could assess the robustness, efficiency and ability to find global minima free of local traps, as all are important qualities to solve a complex optimization problem. Figure 6 shows the Sphere Function which is a smooth, convex benchmark function with a global minimum at (0,0). A basic optimization problem of the parabolic shape has no local minima and is represented by the Sphere function. The figure which shows that the search trajectory approaches the global minimum and does not deviate from it rapidly converged in the Hybrid PSO-GWO algorithm. This constitutes an important result since it demonstrates that the direct gradient is exploited by the algorithm and that simple convex problems, can be quickly solved with minimal computational effort. We show that the Hybrid PSO GWO algorithm exhibits high precision and reliability in performing the Sphere Function, which validates that the Hybrid PSO GWO algorithm can handle straightforward optimization tasks.

Figure 7 shows the Rastrigin Function, a highly multimodal function with a periodic landscape made up of cosine ripples. However, the function has a large number of local minima that are easy-to-catch

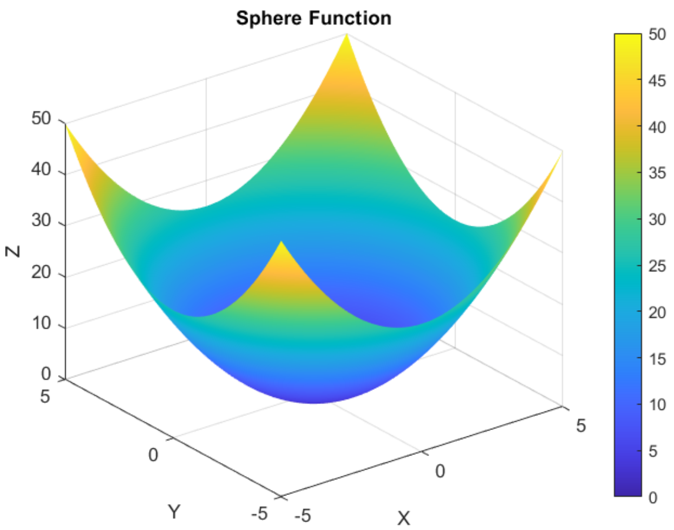

Figure 6: Illustration for the sphere function.

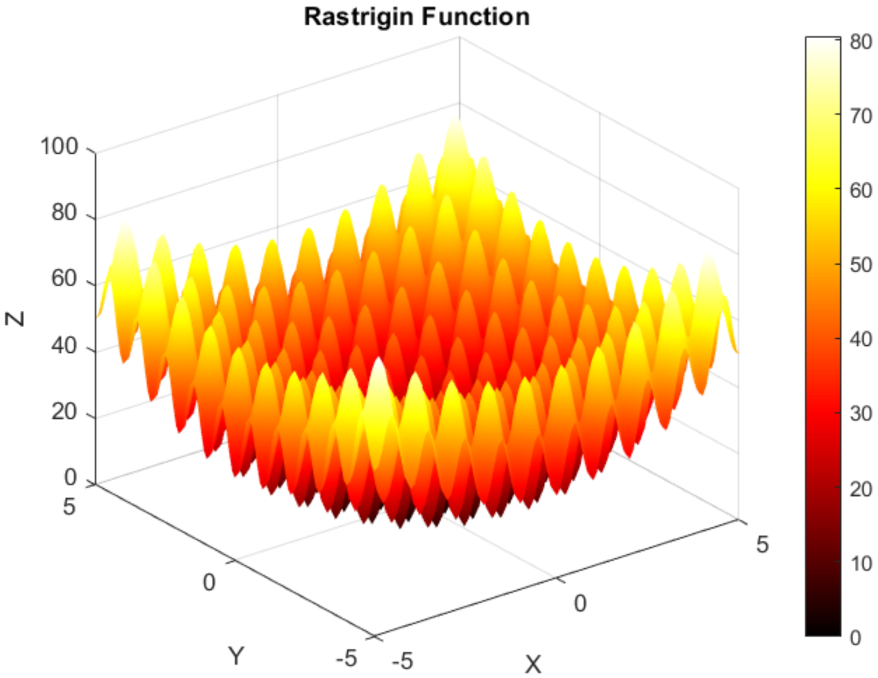

Figure 7: Illustration for the Rastrigin function.

optimization algorithms that do not have strong global search properties. The Hybrid PSO-GWO algorithm successfully manoeuvred the rippled topology and located the global minimum at (0,0) despite the complication of the topology. In the figure, we see how the hybrid algorithm scans the whole function space and only focuses on promising regions. The ability to escape local minima and converge to the global solution determines the robustness of the Hybrid PSO-GWO algorithm for the solution of the multimodal optimization problem.

In Figure 8 we can see the Ackley function which has a very flat outer part and deep valleys leading to a global local minimum in the center. The function has a flat landscape, which is highly deceptive and hard for any optimization algorithm dependent on local gradient information. Nevertheless, the Hybrid PSO GWO algorithm overcame this challenge by its capability to traverse the flat region and converge at the global minimum as shown in the previous Figure 8. Being an effective and adaptable algorithm, it is able to keep the search momentum in flat regions while concentrating on steep gradients at the minimum.

Figure 9 represents the Rosenbrock function, also called the “Banana Function.” The given function has a narrow valley on a ridge with steep, narrow sides; this greatly makes optimization algorithms converge toward the global minimum at (1,1). The Hybrid PSO-GWO follows the valley of the curve and obtains the global minimum. It just serves to show that it can work very well on complex topologies and balance exploration versus exploitation very well. This is because the hybrid approach will prevent the algorithm from drifting out of the valley, rather than leading to its convergence in suboptimal regions of the valley prematurely. The Rosenbrock Function, with this problem being more and more complicated, needed more iterations compared to simpler functions, but Hybrid PSO-GWO performed robustly with this challenging problem.

Figure 10 shows the Griewank Function, a high-dimensional, multimodal problem with many local minima strewn throughout the search space. The mixture of quadratic growth and cosine modulations gives the landscape a hard topography in which finding the global minimum becomes difficult. It is found that the Hybrid PSO-GWO algorithm has very good performance in terms of searching the entire search space and approaching the position corresponding to the global minimum of (0, 0).

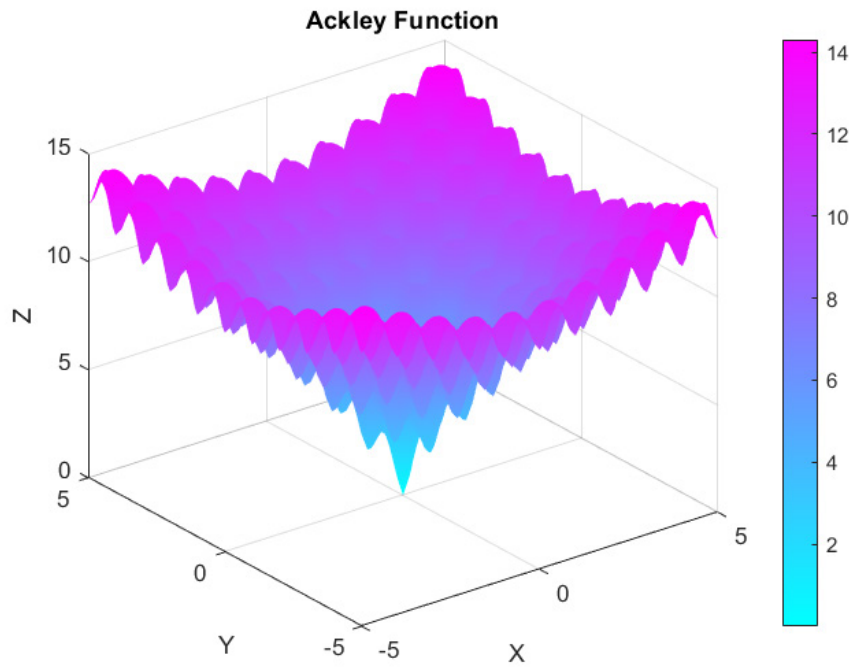

Figure 8: Illustration for the Ackley function.

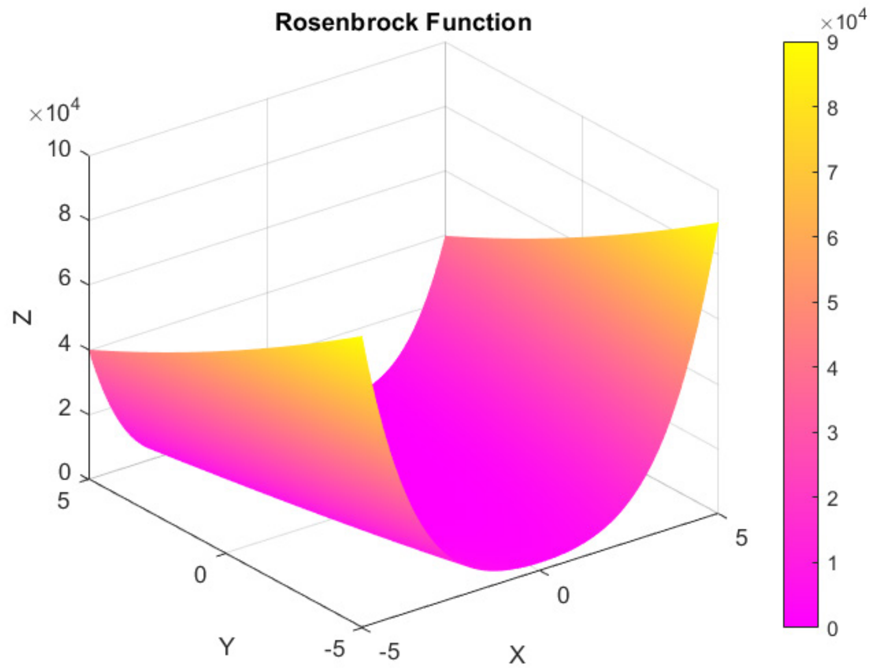

Figure 9: Illustration for the Rosenbrock function.

The algorithm is able to avoid falling into the many local minima and goes on towards an optimal solution, as shown in the figure. This result demonstrates the strength of the hybrid algorithm in solving highly multimodal optimization problems, and in balancing global exploration with local refinement.

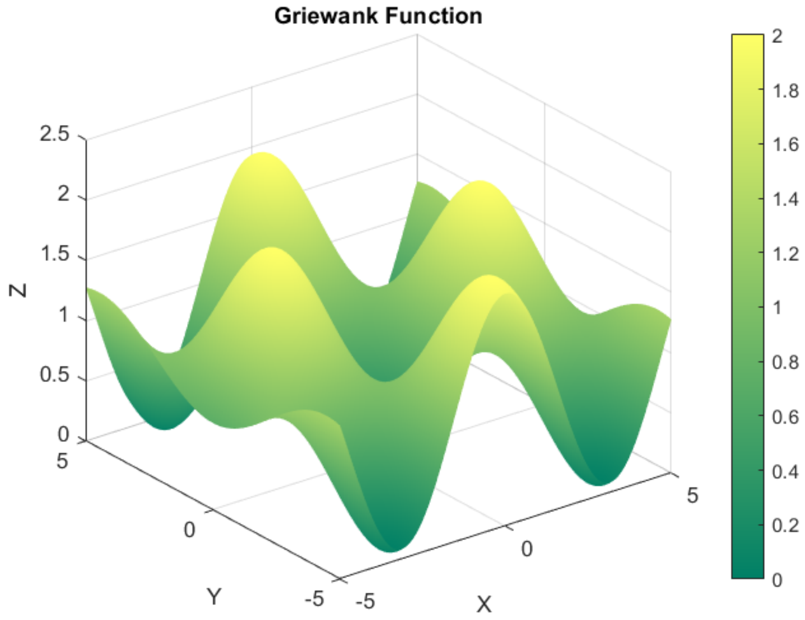

Figure 10: Illustration Griewank function.

8. Conclusion

In this work, we developed and comparatively implemented three optimization algorithms including PSO, GWO, and a Hybrid PSO-GWO algorithm in the context of the optimization of system parameters in order to meet optimised performance in a dynamic control task. To overcome these limitations of standalone algorithms, that is exponential convergence time and premature convergence, we propose the Hybrid PSO-GWO algorithm which combines the exploration strengths of PSO and the exploitation abilities of GWO. Benchmark functions and convergence analysis were used to evaluate each strategy's performance with an emphasis on reducing objective value stability and robustness. We found that the hybrid PSO-GWO algorithm was consistently faster in convergence speed, more accurate and more robust than PSO on its own, and GWO on its own, in line with what we observed in our results. For example, the hybrid algorithm performed better on the more complex benchmark functions, such as Rastrigin, Ackley, Rosenbrock and Griewank, where it encountered local minima and converged to the global minimum. The Hybrid PSO-GWO algorithm (in terms of MSE and final best scores) was better than its standalone counterparts in terms of lower errors as well as better stability during multiple runs. In complex optimization landscapes having multimodal, non-convex and deceptive features, the hybrid algorithm leveraging the global search power of PSO and exploitation ability of GWO can highly conductively searching the complex regions effectively. Furthermore, it was demonstrated that the Hybrid PSO-GWO algorithm is robust and versatile in solving a variety of problems by benchmark evaluations. This ability of the algorithm to maintain the balance between exploration and exploitation led to consistent performance in cases of both simple convex problems and difficult multimodal functions. The findings affirm Hybrid PSO-GWO as a plausible optimization tool for relativity when applied to real-world problems that encompass dynamics and adaptivity of solutions.

References

[1] A. Zaghwan, Y. Amer, M. Efatmaneshnik, and I. Gunawan, "Advancing System of Systems Engineering using intangible value logic measurements from Intellectual Capital Thinking Approach," Heliyon, vol. 10, no. 21, Nov. 2024. doi: 10.1016/j.heliyon.2024.e39814.

[2] M. Habibullah, K. N. Bhumkittipich, N. Mithulananthan, R. Sharma and F. Zare, “Damping Oscillation and Removing Resonance in a RE Based DC Microgrids,” in IEEE Access, vol. 9, pp. 163516-163525, 2021, doi: 10.1109/ACCESS.2021.3135033.

[3] Y. Zeng, B. Liao, Z. Li, C. Hua and S. Li, “A Comprehensive Review of Recent Advances on Intelligence Algorithms and Information Engineering Applications,” in IEEE Access, vol. 12, pp. 135886-135912, 2024, doi: 10.1109/ACCESS.2024.3461756.

[4] N. Basil and H. M. Marhoon, “Selection and evaluation of FOPID criteria for the X-15 adaptive flight control system (AFCS) via Lyapunov candidates: Optimizing trade-offs and critical values using optimization algorithms,” *e-Prime-Advances Electr. Eng. Electron. Energy*, vol. 6, p. 100305, 2023.

[5] F. S. Raheem and N. Basil, “Automation intelligence photovoltaic system for power and voltage issues based on Black Hole Optimization algorithm with FOPID,” *Meas. Sensors*, vol. 25, p. 100640, 2023.

[6] N. Gideon Ude, H. Yskandar and R. Coneth Graham, “A Comprehensive State-of-the-Art Survey on the Transmission Network Expansion Planning Optimization Algorithms,” in IEEE Access, vol. 7, pp. 123158-123181, 2019, doi: 10.1109/ACCESS.2019.2936682.

[7] N. Basil and H. M. Marhoon, “Towards evaluation of the PID criteria based UAVs observation and tracking head within resizable selection by COA algorithm,” *Results Control Optim.*, vol. 12, p. 100279, 2023.

[8] H. M. Marhoon, N. Basil, and A. F. Mohammed, “Medical Defense Nanorobots (MDNRs): a new evaluation and selection of controller criteria for improved disease diagnosis and patient safety using NARMA(L2)-FOP+D(ANFIS)μ – Iλ-based Archimedes Optimization Algorithm,” *Int. J. Inf. Technol.*, 2024, doi: 10.1007/s41870-023-01724-7.

[9] A. F. Mohammed *et al.*, “Selection and Evaluation of Robotic Arm based Conveyor Belts (RACBs) Motions: NARMA (L2)-FO (ANFIS) PD-I based Jaya Optimization Algorithm,” *Int. J. Robot. Control Syst.*, vol. 4, no. 1, pp. 262–290, 2024.

[10] J. Tang, G. Liu and Q. Pan, “A Review on Representative Swarm Intelligence Algorithms for Solving Optimization Problems: Applications and Trends,” in IEEE/CAA Journal of Automatica Sinica, vol. 8, no. 10, pp. 1627-1643, October 2021, doi: 10.1109/JAS.2021.1004129.

[11] N. B. Mohamadwasel and S. Kurnaz, “Implementation of the parallel robot using FOPID with fuzzy type-2 in use social spider optimization algorithm,” *Appl. Nanosci.*, pp. 1–11, 2021, doi: 10.1007/s13204-021-02034-9.

[12] N. Basil and A. Ma’arif, “NB Theory with Bargaining Problem: A New Theory,” *Int. J. Robot. Control Syst.*, vol. 2, no. 3, 2022.

[13] A. Rego, P. L. Ramírez, J. M. Jimenez, and J. Lloret, “Artificial Intelligent System for multimedia services in Smart Home Environments,” Cluster Computing, vol. 25, no. 3, pp. 2085–2105, Jul. 2021. doi: 10.1007/s10586-021-03350-z.

[14] A. F. Mohammed, H. M. Marhoon, N. Basil, and A. Ma’arif, “A New Hybrid Intelligent Fractional Order Proportional Double Derivative+ Integral (FOPDD+ I) Controller with ANFIS Simulated on Automatic Voltage Regulator System,” *Int. J. Robot. Control Syst.*, vol. 4, no. 2, pp. 463–479, 2024.

[15] N. Basil and H. M. Marhoon, “Correction to: selection and evaluation of FOPID criteria for the X-15 adaptive flight control system (AFCS) via Lyapunov candidates: Optimizing trade-offs and critical values using optimization algorithms,” *e-Prime-Advances Electr. Eng. Electron. Energy*, vol. 8, p. 100589, 2024.

[16] N. Basil, B. M. Sabbar, H. M. Marhoon, A. F. Mohammed, and A. Ma’arif, “Systematic Review of Unmanned Aerial Vehicles Control: Challenges, Solutions, and Meta-Heuristic Optimization,” *Int. J. Robot. Control Syst.*, vol. 4, no. 4, pp. 1794–1818, 2024.

[17] A. K. Mishra, S. R. Das, P. K. Ray, R. K. Mallick, A. Mohanty and D. K. Mishra, “PSO-GWO Optimized Fractional Order PID Based Hybrid Shunt Active Power Filter for Power Quality Improvements,” in IEEE Access, vol. 8, pp. 74497-74512, 2020, doi: 10.1109/ACCESS.2020.2988611.

[18] Duan, B., Guo, C. & Liu, H., “A hybrid genetic-particle swarm optimization algorithm for multi-constraint optimization problems”, Soft Computing, vol. 26, pp.11695–11711, 2022, doi: 10.1007/s00500-022-07489-8.

[19] Gharehchopogh, F.S., “An Improved Harris Hawks Optimization Algorithm with Multi-strategy for Community Detection in Social Network”, J Bionic Eng, vol. 20, pp. 1175–1197, 2023, doi: 10.1007/s42235-022-00303-z.

[20] S. Arora, H. Singh, M. Sharma, S. Sharma and P. Anand, “A New Hybrid Algorithm Based on Grey Wolf Optimization and Crow Search Algorithm for Unconstrained Function Optimization and Feature Selection,” in IEEE Access, vol. 7, pp. 26343-26361, 2019, doi: 10.1109/ACCESS.2019.2897325.

[21] Abbas, M., Alshehri, M. A., & Barnawi, A. B., “Potential Contribution of the Grey Wolf Optimization Algorithm in Reducing Active Power Losses in Electrical Power Systems”, Applied Sciences, vol. 12, no. 12, p. 6177, 2022, doi: 10.3390/app12126177.

[22] Yu, M., Xu, J., Liang, W. et al., “Improved multi-strategy adaptive Grey Wolf Optimization for practical engineering applications and high-dimensional problem solving”, Artif. Intell. Rev., vol.57, p.277, 2024, doi: 10.1007/s10462-024-10821-3.

[23] Y. Zhang and Y. Cai, “Adaptive dynamic self-learning grey wolf optimization algorithm for solving global optimization problems and engineering problems,” Mathematical Biosciences and Engineering, vol. 21, no. 3, pp. 3910–3943, 2024. doi: 10.3934/mbe.2024174.

[24] Yuan, P., Zhang, T., Yao, L., Lu, Y., & Zhuang, W., “A Hybrid Golden Jackal Optimization and Golden Sine Algorithm with Dynamic Lens-Imaging Learning for Global Optimization Problems”, Applied Sciences, vol. 12, no. 19, p. 9709, 2022, doi: 10.3390/app12199709.

[25] Papazoglou, G., & Biskas, P., “Review and Comparison of Genetic Algorithm and Particle Swarm Optimization in the Optimal Power Flow Problem”, Energies, vol. 16, no. 3, p.1152, 2023, doi: 10.3390/en16031152.