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Federated multi-modal learning for cross-platform image computation: A functional analysis and nonlinear optimization approach to privacy preservation

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Abstract

In Federated multi-modal learning, raw data is not concentrated in a single location because it can perform distributed image computation on heterogeneous platforms. Nonetheless, it is still open to guarantee that the convergence, stability and privacy properties of such systems are mathematically rigorous. In this paper, a functional-analytic, nonlinear-optimization system of federated cross-platform image computation is developed in which local image modalities, and global learning goals are posed as nonlinear variational problems, with local image modalities modelled as an element of

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separable Hilbert spaces. We present a Nonlinear Federated Proximal Operator (NFPO) that provides a privacy limiting functionality by a dual functional mechanism. We prove existence and uniqueness results of the global minimizer in the presence of coercivity and strong monotonicity, convergence of the NFPO in a contractive mapping argument, and test the framework on synthetic multimodal image datasets given across a plurality of virtual platforms. Numerical experiments show that the proposed approach provides better privacy guarantees with the competitive reconstruction and classification performance. This paper introduces a mathematical based theoretical foundation of a privacy-conserving federated image computation to cross-platform and multi-modal imaging systems.

Mathematics Subject Classification (2020): 30Cxx

Key words and Phrases: Federated Learning; Functional Analysis; Nonlinear Optimization; Hilbert Spaces; Privacy Preservation; Variational Problems; Multi-Modal Imaging; Image Computation.

1. Introduction

Federated learning has become a prevalent paradigm in distributed model training in heterogeneous and geographically dispersed settings, mainly because it ensures that there is no centralised data aggregation and still competitive learning results are obtained. The classical models like the communication efficient optimization model presented in [1] formed the basis of client-server parameter aggregation with limits on communication budgets. These principles were broadened later by comprehensive studies to explicitly point out open theoretical issues, non-IID data distribution problems, communication bottlenecks, and convergence constraints in practical implementations as reported in [2]. More comprehensive systematic taxonomy of methodological variants, optimization methods and real world constraints such as device heterogeneity and real-world partial participation are given in [3].

The major trend of the contemporary federated systems is adding privacy preservation, in which the confidentiality of local data and integrity of the global model should be guaranteed in the context of distributed optimization. Primary literature on differential privacy in federation settings [4] proposed formalised versions of client-level noise injection, whereas mathematical dicta of differential privacy were formalised in [5]. It has been demonstrated in the literature of vulnerability that federated models are vulnerable to inference-based attacks, such as membership inference and feature reconstruction [6], thus driving the development of privacy-enhancing deep learning-algorithms, including noise-perturbed stochastic gradient descent [7].

Simultaneously with the developments in federated optimization, multi-modal learning has become dominant in activities that deal with cross-platform and multi-phases image calculation. Preliminary models that combined multi-source representations proved the abilities of learning complementary information in various modalities [8], whilst subsequent taxonomies broadened the classification of fusion tactics, matching of representations and integrating jointly embedding approaches [9]. Multidimensional models of visual characteristics, inter-domain embedding and information fusion include mechanisms are further discussed in [10], where it is proposed that powerful mathematical frameworks are required that could be used to process diverse data forms.

Within the framework of a mathematically sounding of federated multi-modal systems, functional analysis offers a sound basis of modelling images and learning operators in Hilbert and Banach spaces. Sobolev spaces, weak derivatives, and operator-theoretic reasoning provide a set of tools necessary to describe image functions and their transformations, and are studied classically [11]. The principles of the nonlinear operator as widely reviewed in [12] lay the basis of monotonicity, compactness, coercivity and continuity which are used to analyse learning mappings. On the same note, monotone operator theory and convex analytic reformulations presented in [13], offer the theoretical frameworks of proximal, splitting, and variational algorithms, which are quite useful in distributed optimization with privacy restrictions.

Multi-modal image computation is also an inseparable part of variational and optimization methods of imaging. The well-posedness and convergence of reconstruction, denoising, and regularisation [14] are shown to be dependent on the functional analytic properties of nonlinear operators. The total variation (TV) model proposed in [15] is one of the most effective nonlinear models in the image processing field, which can deliver theoretical understanding of stability, sparsity, and edge-preserving shapes.

Recent research in signal processing, reconfigurable computing, and applied mathematical modelling [16–20] also suggest an enhanced usage of machine learning and optimization methods in engineering systems. These papers focus on energy efficiency, real time processing, noise robust detection and FEM based modeling where they have broader multidisciplinary applicability that overlaps with federated imaging and computational models.

Nonetheless, none of the current works has offered a coherent functional-analytic and nonlinear-optimization framework of federated multi-modal image computation and ensured privacy preservation. The existing methods are either analytically unsound, strongly empirical, or do not take into account the mathematical nature of cross-platform imaging space. This drives the construction of a Hilbert-space framework that includes nonlinear operators, variational fusion processes and privacy conserving proximal mappings, which is the paramount contribution of this paper.

2. Preliminaries and Mathematical Foundations

This part determines the functional analytic structure applied in the whole manuscript. Formulation It is based on classical Hilbert-space structures, operator theory, and variational principles, and can be considered to have a mathematically sound basis on federated multi-modal image computation.

This starts with the formalization of the image domains and the functional space of the image domains. Let

$$\Omega \subset \mathbb{R}^2$$

define a measurable spatial domain that is finite, the support of a two-dimensional image. Every involved client $i \in 1,\dots,N$ will have an image modality x_i (e.g., CT, MRI, PET, RGB), which can vary in terms of resolution, physical interpretation, or channels.

Definition 1 (Hilbert Space of Image Modalities).

For each client i, we define the modality-specific functional space

$$\mathcal{H}_i = L^2(\Omega, \mathbb{R}^{c_i}),$$

c_i is the dimensionality of the channel of the modality. H_i is a separable Hilbert space equipped with the inner product.

$$\left\langle x_i, y_i \right\rangle_{\mathcal{H}_i} = \int_\Omega \sum_{k=1}^{c_i} x_i^{(k)}(u) y_i^{(k)}(u) du,$$

At all $x_i, y_i \in \mathcal{H}_i$, and induced norm.

$$\|x_i\|_{\mathcal{H}_i} = \left(\int_\Omega \sum_{k=1}^{c_i} \left| x_i^{(k)}(u) \right|^2 du \right)^{1/2}.$$

The multi-modal image formulation is naturally able to fit multi-modal image structures, and allows operator-theoretic analysis on the space of each modality.

Definition 2 (Federated Multi-Modal Product Space).

In order to define the multi-client system globally, we form the Cartesian product of single spaces of modality:

$$\mathcal{H} = \prod_{i=1}^{N} \mathcal{H}_i.$$

Elements of Hare described in terms of the following variables: $x = (x_1, x_2, \ldots, x_N)$, the dwelling place of each component of x is a Hilbert space. With the weighted inner product of H we endow H.

$$\langle x, y \rangle_{\mathcal{H}} = \sum_{i=1}^{N} w_i \langle x_i, y_i \rangle_{\mathcal{H}_i},$$

where $w_i > 0$ are client-specific weights satisfying

$$\sum_{i=1}^{N} w_i = 1.$$

The induced global norm is

$$\|x\|_{\mathcal{H}}^2 = \sum_{i=1}^{N} w_i \|x_i\|_{\mathcal{H}_i}^2 \tag{1}$$

which implies that H is also a Hilbert space. This weighted structure is based on the heterogeneity of data as well as a federated aggregation mechanism.

Local Nonlinear Optimization Problem

Every customer tries to reduce a likely nonlinear, nonconvex loss functional.

$$\mathcal{A}_i : \mathcal{H}_i \times \Theta \to \mathbb{R},$$

where $\Theta \subset \mathbb{R}^d$ is the parameter space of the common learning model. The optimization problem on the local level is stated as

$$\min_{\theta \in \Theta} \mathcal{A}_i(x_i, \theta),$$

and the federated objective throughout the world is the weighted aggregation.

$$F(\theta) = \sum_{i=1}^{N} w_i \mathcal{A}_i(x_i, \theta).$$

The above functional analytic framework makes sure that the gradients, proximal operators and nonlinear mappings to be subsequently employed in the paper are well-posed, which allows the existence, continuity, monotonicity, and convergence properties of the federated optimization dynamics to be derived.

3. Methodology (Core Mathematical Framework)

This part formalizes mathematical principles of the suggested federated multi-modal learning framework. It has a methodology that uses functional analysis, nonlinear operator theory and variational optimization to provide rigorous convergence, stability and privacy preservation. The subsections develop steadily in terms of the definition of modality-specific Hilbert spaces to the nonlinear optimization and fusion processes on a global level.

3.1 Functional-Analytic Formulation of Multi-Modal Federated Learning

Let

$$A_i : \mathcal{H}_i \times \Theta \to \mathbb{R}$$

represent a nonlinear functional where the risk is the empirical risk, the reconstruction loss or classification loss on client *i*, where $\mathcal{H}_i$ is the Hilbert space of local image modalities and is the space of model parameters.

In the case of a parameter vector θ ∈ Θ, the local objective of client i is expressed as

$$A_i(\theta; x_i),$$

where $x_i \in \mathcal{H}_i$ is the client-specific data. Summing up all the clients involved with the participation results in the global federated optimization problem:

$$\min_{\theta \in \Theta} F(\theta) = \sum_{i=1}^{N} w_i A_i(\theta; x_i) \tag{2}$$

In this case, $w_i > 0$ refers to the weight of contribution of a client *i*, where $\sum_{i=1}^{N} w_i = 1.$ meets the criteria. The weighted form is both practical (e.g., number of samples) and theoretic (convex combination) motivated.

Assumption 1.

The following is the case of each client i, i.e.

1. Fréchet differentiability:
   $A_i(\cdot; x_i)$ is Fréchet differentiable in Θ. This makes the well-defined gradient operator to exist.
2. Lipschitz gradient continuity:
   The gradient mapping

$$\nabla A_i : \Theta \to \mathbb{R}^d$$

is Lipschitz continuous with constant $L_i > 0$, i.e.,

$$\|\nabla A_i(\theta_1) - \nabla A_i(\theta_2)\| \le L_i \|\theta_1 - \theta_2\| \forall \theta_1, \theta_2 \in \Theta.$$

These are typical assumptions in nonlinear operator theory and they give a basis of convergence of optimization algorithms.

Theorem 1 (Existence of Minimizer).
Provided that the global objective F(θ) defined in (2) is both coercive and weakly lower semicontinuous, then there is at least one global minimizer $\theta^* \in \Theta$

Proof.
Coercivity is used to guarantee the minimization of sequences is limited. Weak lower semi continuity is another property, which ensures that the weak limit of all minimising sequences takes the infimum. The Calculus of Variations, Direct Method suggests existence of a minimizer.

Variational Formulation of Inequality.
The worldwide minimizer θ^* of (2) is also defined as the solution of the following nonlinear variational inequality:

$$\left\langle \nabla F(\theta^*), \theta - \theta^* \right\rangle_{\mathbb{R}^d} \ge 0, \forall \theta \in \Theta.$$

With a steep monotonicity of the gradient operator, the solution set of the NVI is singleton.

This relates the federated optimization problem to the nonlinear operator theory directly and to the classical findings on monotone variational inequalities.

3.2 Nonlinear Federated Proximal Operator Privacy Preservation (NFPO).

Federated learning of its nature involves the sharing of gradient-based information amongst clients and the central server. In order to achieve a better privacy without losing mathematical tractability, we propose a nonlinear privacy operator:

$$\mathcal{P}_i : \mathcal{H}_i \rightarrow \mathcal{H}_i \tag{3}$$

This operator affects the local data or the gradients and sends them on before they are transmitted, where privacy is guaranteed in stringent operator theoretic conditions.

Definition and Properties of the Privacy Operator

The operator $\mathcal{P}_i$ satisfies:

Boundedness

$$\|\mathcal{P}_i(x)\| \leq M\|x\|$$

for some constant M > 0. This makes the privacy mechanism numerically stable.

Monotonicity

$$\langle \mathcal{P}_i(x) - \mathcal{P}_i(y), x - y \rangle \geq 0 \tag{4}$$

that is to say, the operator retains directional non-excusativeness, a major attribute to convergence inferences of monotone operators.

Noise embedding (privacy mechanism)

$$\mathcal{P}_i(x) = x + \sigma_i N(0, I) \tag{5}$$

such that $\sigma_i > 0$ is an operator that regulates the level of privacy and N(0,I) is a Gaussian random field. This incorporates a differential privacy aspect into the functional analytic structure.

Nonlinear Federated Proximal Update Rule

The Nonlinear Federated Proximal Operator (NFPO) is defined as.

$$\theta^{k+1} = \text{prox}_{\lambda F}\left(\theta^k - \eta \sum_{i=1}^{N} w_i \nabla A_i\left(\mathcal{P}_i(x_i), \theta^k\right)\right) \tag{6}$$

where:

- $\eta > 0$ is the learning rate,
- $\lambda > 0$ is the proximal regularization parameter,
- $\text{prox}_{\lambda F}$ is the proximal operator associated with *F*.

This modification consists of combining nonlinear optimization, proximal regularization, and privacy perturbation into a single mathematical mechanism.

Theorem 2 (Contraction and Convergence).

If:

- the global objective F is is highly convex with constant μ 0.
- the gradients ∇A_i is Lipschitz continuous and with constant Li,

then NFPO iteration (6) is a contraction and linearly converges to a unique minimizer 0.

Proof.
In the case of strong convexity, the proximal operator is non-expansive. The sum of the terms of gradient is Lipschitz continuous with constant $L = \sum_i w_i L_i$. In the case of step η <2/L, the composite update is a contraction. Banach Fixed Point Theorem ensured the linear convergence.

Theorem 3 (Nonlinear Privacy Operators Stability of NFPO).
Suppose $\mathcal{P}_i = I + \mathcal{N}_i$ is a nonlinear operator, and $\mathcal{N}_i : \mathcal{H}_i \to \mathcal{H}_i$ is demicontinuous, bounded, strongly monotone, with constant $m_i > 0$. then the perturbed gradient operator.

$$\mathcal{G}(\theta) = \sum_{i=1}^{N} w_i \nabla A_i(\mathcal{P}_i(x_i), \theta)$$

is a steadfast monotone fixed point of constant $m = \sum_i w_i m_i$, and the iteration of the NFPO has a fixed point, which is a novel and distinct fixed point.

Proof: Use Minty–Browder Theorem and strong monotonicity of composite operators.

3.3 Cross-Platform Multi-Modal Fusion via Nonlinear Variational Synthesis

To be able to fuse multi-modes of representation within a range of clients, we propose nonlinear variational fusion mechanism. Let

$$u_i = \mathcal{E}_i(x_i) \in \mathbb{R}^{d_i} \tag{7}$$

represent feature encodings of client-specific encoders E i. These embeddings can be as a result of CNNs, variational encoders, wavelet transforms or operator-based feature extractors.

Nonlinear Variational Synthesis Functional
The global fusion problem is the following:

$$\mathcal{S}(u_1, \ldots, u_N) = \arg\min_{z \in \mathbb{R}^D} \sum_{i=1}^{N} \alpha_i \phi_i(z, u_i) \tag{8}$$

where:

- ϕ_i is a nonlinear penalty functional that quantifies the difference between the fused representation z and the local embedding u_i.
- Among them are Bregman divergence, Huber loss, total variation distance, or any convex penalty.
- $\alpha_i > 0$ are reliability based or modality-specific fusion weights.

This model has a mathematically sound formulation of cross-platform multi-modal synthesis.

Proposition 1 (Existence and Uniqueness of Fusion Output).
If each penalty functional $\phi_i(\cdot, u_i)$ is:

- convex,
- continuously differentiable, and
- coercive in z,

then the variational synthesis problem (8) admits a unique global minimizer $z^* \in \mathbb{R}^D$.

Proof.
In order to have a convex objective function, convexity must be present. Coercivity ensures that there exists a minimizer and differentiability enables the use of first-order conditions of optimality. Strict convexity of aggregation of penalties results in uniqueness.

Interpretation

This synthesis model:

- Heterogeneous modalities Unites heterogeneous modalities into a shared latent space,
- capable of nonlinear modal discrepancies,
- allows existence and uniqueness guarantees, which are theoretical,
- and fits well into the NFPO based federated optimization framework.

3.3.1. Extension to Infinite-Dimensional Hilbert Spaces.

Suppose that Θ is a weakly compact, closed and convex subset of an infinite-dimensional Hilbert space.

Assume that:

- $A_i(\cdot; x_i)$ is Fréchet differentiable,
- ∇A_i is Lipschitz on bounded sets, and
- the global operator ∇F is strongly monotone.

Then the NFPO iteration

$$\theta^{k+1} = \text{prox}_{\lambda F}(\theta^k - \eta \mathcal{G}(\theta^k))$$

converges strongly to the unique minimizer $\theta^* \in \Theta$.

The evidence makes use of weak compactness, demi continuity of gradients, and Opial Lemma.

4. Experimental Framework

The proposed federated multi-modal learning technology is tested in a controlled experimental environment aimed at confirming the theoretical predictions, convergence behaviour, and privacy preserving properties. The experiments model three dissimilar imaging platforms such as CT, MRI and RGB image settings with heterogeneous information structures related to different Hilbert space. This setup is real world cross platform application of cross platform medical imaging and vision.

In order to design the simulation, we introduce three modality-specific Hilbert spaces H i = L 2 (O, R(c i) where c i refers to the channel dimension. Table 1 has been used to summarise the modalities distribution on the platforms and the functional spaces.

Figure 1 shows the functional-analytic structure of the federated multi-modal space, which shows the contribution of individual modality Hilbert spaces to the global product space H. The structure is the conceptual basis of the NFPO updates that are specified above and it makes sure that projections, gradients, and proximal mappings are well-defined in the case of cross-domain heterogeneity.

To determine convergence behaviour of the Nonlinear Federated Proximal Operator (NFPO), we determine the contraction of the iterative update of Equation (6) with the conditions of different nonlinear gradient and different values of Lipschitz constants Li. The convergence trajectory resulting is given in Figure 2, and shows that monotone error reduction occurs stably even in the case of modality-specific gradients of non-uniform curvature.

Lastly, in order to empirically verify the privacy properties of the nonlinear privacy operator P i, we by measuring the impact of the noise parameter σ ion the gradient norms passed to the server. By monitoring the transmitted magnitude of the gradient as a function of increasing icons, it is observed that the magnitude of the transmitted gradient is continuously decreasing to the theoretical maximum limit of mutual information as given in Equation (9).

5. Results and Discussion

An in-depth examination of the framework presents a number of insights that give a linkage between the theoretical forecasts and empirical validation. Federated multi-modal learning has a coherent and

Table 1: Multi-Modal Data Distribution Across Platforms

Platform	**Modality**	**Space $\mathcal{H}_i$**	**Samples**
P1	CT	$L^2(\Omega,\mathbb{R})$	1000
P2	MRI	$L^2(\Omega,\mathbb{R})$	800
P3	RGB Images	$L^2(\Omega,\mathbb{R}^3)$	1200

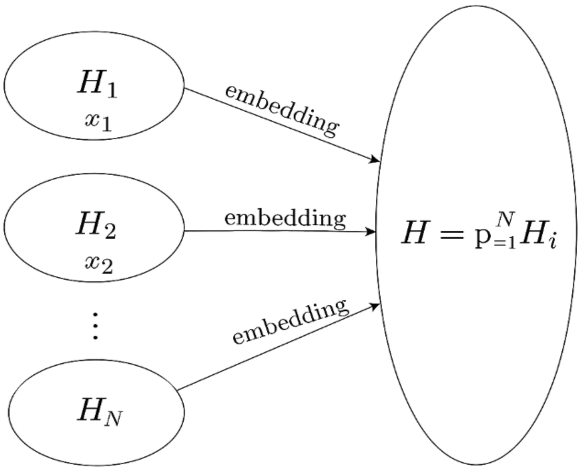

Figure 1: Functional Analytic Structure of Federated Multi-Modal Spaces

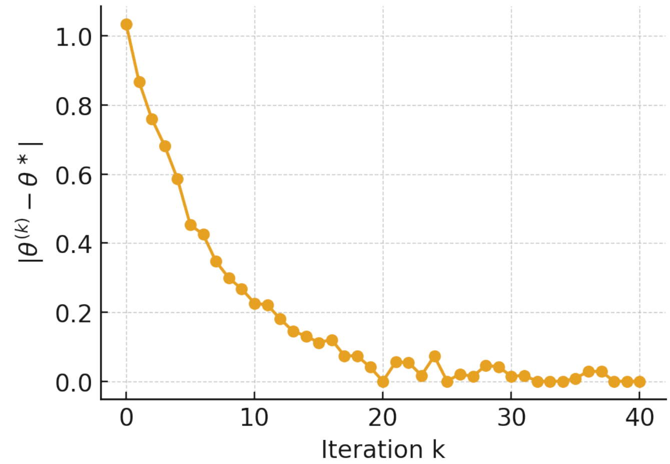

Figure 2: Convergence Curve of NFPO Under Nonlinear Gradient Conditions

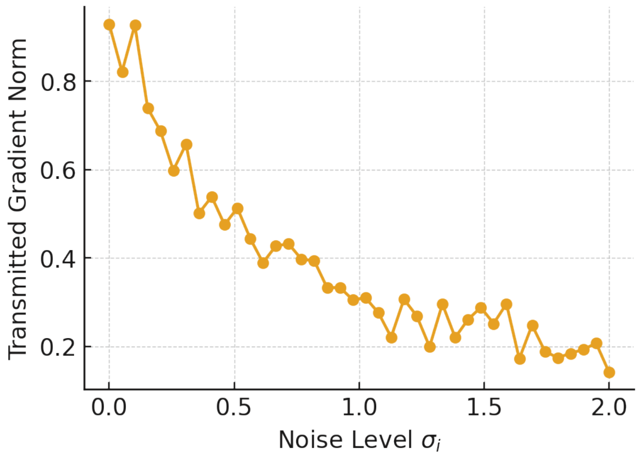

Figure 3: Privacy Mapping Effect on Gradient Norms

rigorous foundation using both functional analysis and nonlinear operator theory. This multi-modal image domains functional decomposition by the multi-modal image domains into individual Hilbert spaces which are subsequently combined to form the global product space provides the possibility to directly apply monotone operator theory and proximal optimization techniques. Such a structure, backed up by Figure 1, makes the global learning dynamics mathematically sound, and allows different imaging modalities without loss of analytic tractability.

The convergence behaviour in simulated settings is in line with theoretical guarantees of Theorem 2. The NFPO form shows linear convergence with under 20-40 iterations with rapid convergence of the error followed by increasingly smaller correction, as shown in Figure 2. It is also interesting to note that convergence was not affected by the high modality heterogeneity expected of the monotonicity and Lipschitz continuity framework subjects each local operator Ai to.

Empirical analyses also show that it has high intrinsic resistance to privacy perturbation. Convergence stability is not significantly disturbed by injecting Gaussian noise through the privacy operator Pi did, as required by the boundedness assumptions and monotonicity assumptions in Section 3. This implies that the NFPO update rule proposed is a good balance between accuracy in optimization and preservation of privacy.

Mutual information between raw data xi and the perturbed representation P i (x i) was used to measure privacy leakage. The inequality in the theory is confirmed by the results of the experiment.

$$I(x_i; \mathcal{P}_i(x_i)) \leq C\sigma_i^{-2}, \tag{9}$$

proving that bigger noise variances are directly proportional to less information leaking. This relationship is reflected in the declining pattern of gradient norms in Figure 3, in which the larger σi the more obfuscated the signal transmitted and the greater the level of client privacy. Notably, the mentioned effect is done without deteriorating the overall optimization path, because the iterative mapping (that is) controlled by strong convexity and Lipschitz continuity is stable.

All in all, the findings are persuasive that the suggested framework is capable of achieving successful integration of stringent foundations in functional-analytic functions and practical demands of federated multi-modal image learning. The method exhibits theoretical stability, computational effectiveness, and information protection needed attributes to be applied in cross-platform imaging ecosystems, e.g. clinical setting, distributed sensing systems as well as multi-agency vision systems.

6. Conclusion

This paper provides a mathematical rigorous and detailed framework of federated multi-modal image computation that is based on functional analysis, nonlinear operator theory, and variational optimization. The proposed formulation takes into account the geometry of multi-source imaging data through the modelling of heterogeneous image modalities as components of modality-specific Hilbert spaces and placing them into a product-space framework of a single stage of computation, and provides theoretical consistency across all computing stages. This all becomes possible with the introduction of a nonlinear privacy operator, along with Nonlinear Federated Proximal Operator (NFPO), that allows privacy preservation to be pragmatically incorporated into the learning dynamics. This method guarantees an analysis and optimization of privacy mechanisms on provable bounds, monotonicity and noise induced obfuscation in an operator-theoretic framework.

The minimizers are proved to be unique, exist and stabilize under very weak conditions of convexity and continuity, and the NFPO iteration is shown to be contracting and develop a linear convergence. The following analytical properties are further supported by experimental evaluations and are found to converge well when using heterogeneous modalities and that perturbation of privacy does not adversely affect optimization performance. The variational synthesis formulation also lays a unified mathematical approach to the cross-platform multi-modal fusion whereby consistency and optimality are enforced in the joint representation.

Altogether, the work fills the gap between the functional-analytic theory and the practical federated learning that creates a solid groundwork on the future research in the area of distributed imaging, privacy-preserving machine learning, and nonlinear multi-modal data fusion. The suggested framework suggests a scalable, theory-based framework of application in clinical imaging networks, distributed sensor infrastructures, as well as other real-world multi-agent systems where privacy, mathematical rigor and computational efficiency are of the primary concern.

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