



Quantum calculus-based nonlinear analysis and neural approximation of fuzzy partial differential models in fluid dynamics

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Abstract

This paper constructs a firm quantum calculus based nonlinear analytical framework of fuzzy partial differentiable models that are encountered in fluid dynamics. The fuzzy nonlinear equation governing is expressed in terms of a q-time derivative operator and reduced to a corresponding operator equation in a Banach space. The locality of solutions and existence are proved through contraction mapping principles on the appropriate Lipschitz and coerciveness conditions. The exponential stability is obtained using a Lyapunov functional method and provides explicit decay rates which are proportional to viscosity and nonlinear growth parameters. In an attempt to improve computationally the tractability of computations and maintain analytical properties, a residual-based scheme of neural approximations is proposed. A nonlinear residual functional is then formulated so that it would be consistent with the governing fuzzy quantum PDE and convergence estimates are derived in the correct Sobolev norms. The contribution of the quantum parameter is calculated in a systematic manner and its impact on the stability rates, minimization of the residual and damping behaviour have been identified.

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The theoretical findings are confirmed by the numerical experiments that show that neural approximation error decays polynomially, fuzzy uncertainty propagation is bounded by 8 levels of the 8-level, and the optimization dynamics is stable on a variety of quantum parameter space. The suggested framework combines the quantum operator theory, nonlinear functional analysis, and neural approximation into a consistent approach to uncertain modelling of nonlinear fuzzy fluid flow systems.

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Key words and Phrases: Quantum calculus; Fuzzy partial differential equations; Banach fixed-point theory; Lyapunov stability analysis; Physics-informed neural networks.

1. Introduction

The nonlinear fluid dynamics models are dictated by partial differential equations (PDEs) which are often very nonlinear, convectively dominated and sensitive to uncertain parameters. Measurement imprecision and epistemic uncertainty are too important in practise engineering and physical systems to be tackled satisfactorily in classical deterministic formulations. Fuzzy partial differential equations (FPDEs) offer a strict mathematical paradigm of the explicit introduction of uncertainty into the system parameters and state variables [1–4,11,23]. These have been used to solve nonlinear boundary value problems, fractional fuzzy systems and modelling of flows in the presence of uncertainty [3,6,8,11].

Nonlinear FPDEs are difficult to analyse analytically since they exhibit nonlinearity in operators, 2D coupling at 2 levels of 3D coupling, in addition to non-convex solution structures [4,8,23]. Lipschitz and compactness conditions have been demonstrated necessary to establish existence and uniqueness results of fuzzy PDEs in Banach spaces [23] and the Lyapunov-based techniques have been developed to investigate stability in fuzzy fractional systems [24]. The analytical methods that have been investigated to solve nonlinear fuzzy PDEs are residual-based and transform methods [6,8,11].

The quantum calculus (q-calculus) is a generalised differential calculus that removes the classical processes of limits, and allows discrete-continuous quasi-hybrid modelling of dynamical systems [12,14]. q-derivative operator has been used to integro-differential equations and nonlinear evolution problems which provide greater flexibility in regulating time-scaling and damping processes [12,14]. Recent developments in quantum algorithms in the solution of the equation of state substantially show the computational significance of q-based models in nonlinear system analysis [7,9]. The inclusion of quantum operators into uncertainties fluid systems, brings in another parameter of structure that is able to control stability properties and convergence properties.

Simultaneously, neural network-based approximation methods have become an effective way of solving nonlinear PDEs. Physics-informed neural networks (PINNs) and residual minimization models incorporate the governing equations into the data-driven but still physically consistent solutions strategies through direct use of governing equations as the training goals [16,17,19]. Generalizations of fuzzy and uncertainty-sensitive systems have shown that neural models can solve fuzzy differential equations with interval consistency [21,25]. Other engineering and computational applications of neural optimization methods have also been successfully implemented in acoustics, thermo-mechanical systems, adaptive control, and multimodal modelling [5,10,13,15,18,20,22] and the expanding role of AI-based methods in the analysis of complex systems.

Inspired by these advances, this paper combines q-calculus based nonlinear operator theory with neural residual minimization to build a mathematically sound framework of the fuzzy fluid flow systems. It rewrites the governing fuzzy nonlinear equation with a q-time derivative operator and studies it in a Banach space context, and is based on Lax foundations of existence theory of fuzzy PDEs [23] and Lyapunov stability [24]. An extended version of neural PDE approaches [16,17,19,21,25] is presented by introducing a residual local functional that is consistent with the fuzzy quantum operator framework.

The proposed methodology brings together fuzzy operator theory, quantum calculus, nonlinear functional analysis, and neural approximation under a single framework of analysis in order to offer a consistency of stability and preservation of uncertainty in nonlinear fuzzy fluid dynamics modelling.

2. Nonlinear Analytical Framework

2.1 Mathematical Formulation of the Fuzzy Quantum Fluid Model

The article takes into account a nonlinear fuzzy partial derivative (PDE) model in fluid mechanics that is acted by a quantum time-derivative operator in a bounded spatial domain $\Omega \subset \mathbb{R}^d$ with a sufficiently smooth boundary $\partial\Omega$. The ruling equation can be written as.

$$D_q^t \tilde{u}(x, t) + \mathcal{N}(\tilde{u}(x, t)) = \nu \Delta \tilde{u}(x, t), \quad x \in \Omega, \quad t > 0,$$

where D_q^t is the q -time derivative, $\mathcal{N}(\tilde{u})$ is the nonlinear convection operator (typically of Burgers or Navier-Stokes type), and $\nu > 0$ is the viscosity parameter. The state variable is of fuzzy type and is expressed in terms of α -level as given below.

$$\tilde{u}(x, t) = [u^-(x, t, \alpha), u^+(x, t, \alpha)], \quad \alpha \in [0, 1].$$

This α -level decomposition reduces the fuzzy PDE to a system of coupled deterministic PDEs with uncertainty bounds. The system is complemented with initial condition.

$$\tilde{u}(x, 0) = \tilde{u}_0(x),$$

and homogeneous Dirichlet boundary condition

$$\tilde{u}(x, t)|_{\partial\Omega} = 0.$$

The quantum derivative is defined by

$$D_q f(t) = \frac{f(qt) - f(t)}{(q-1)t}, \quad q \neq 1,$$

which tends to the classical derivative in the case $q \rightarrow 1$. As opposed to the classical differentiation, q -calculus presents a discrete scaling factor that adjusts dynamics of temporal evolution and becomes more in control of the damping behavior.

To analyse it rigorously we make the following hypotheses:

(A1) Regularity of Domain

Assume that $\Omega \subset \mathbb{R}^d$ is a bounded domain whose boundary $\partial\Omega$ is smooth.

(A2) Nonlinear Operator Condition

The convection operator

$$N : H_0^1(\Omega) \rightarrow L^2(\Omega)$$

satisfies the Lipschitz condition

$$\|N(u) - N(v)\|_{L^2} \leq L_1 \|u - v\|_{H_0^1}$$

for some constant $L_1 > 0$.

(A3) Poincaré Inequality

There exists $\lambda_1 > 0$ such that

$$\|\nabla u\|_{L^2}^2 \geq \lambda_1 \|u\|_{L^2}^2, \forall u \in H_0^1(\Omega).$$

Such assumptions guarantee the operator mappings which are clear in the framework of Banach space. Figure 1 depicts the conceptual flow of the development of models that use fuzzy PDE modeling to quantum operator modeling: the α -level decomposition is linked to the applications of q-operators and representation of the nonlinear system through structured pipeline. It is described in the interval form, and then fuzziness is converted by q-calculus to nonlinear operator system to apply functional analysis, as depicted in Figure 1. The formulation allows treating it rigorously yet has the ability to represent uncertainty.

2.2 Operator Reformulation and Fixed-Point Analysis

In order to examine well-posedness, the fuzzy quantum PDE is reformulated to an equivalent integral equation by the use of the q-integral operator.

$$\int_0^t f(\tau) d_q \tau = (1-q)t \sum_{k=0}^{\infty} q^k f(q^k t).$$

The derivation of the governing equation by the use of q-integration gives

$$\tilde{u}(t) = \tilde{u}_0 + \int_0^t (v\Delta\tilde{u} - \mathcal{N}(\tilde{u})) d_q \tau.$$

Define the nonlinear operator

$$\mathcal{T}(\tilde{u}) = \tilde{u}_0 + \int_0^t (v\Delta\tilde{u} - \mathcal{N}(\tilde{u})) d_q \tau.$$

Suppose that $X = C([0, T]; H_0^1(\Omega))$ is a Banach space with norm.

$$\|\tilde{u}\|_X = \sup_{t \in [0, T]} \|\tilde{u}(t)\|_{H_0^1(\Omega)}.$$

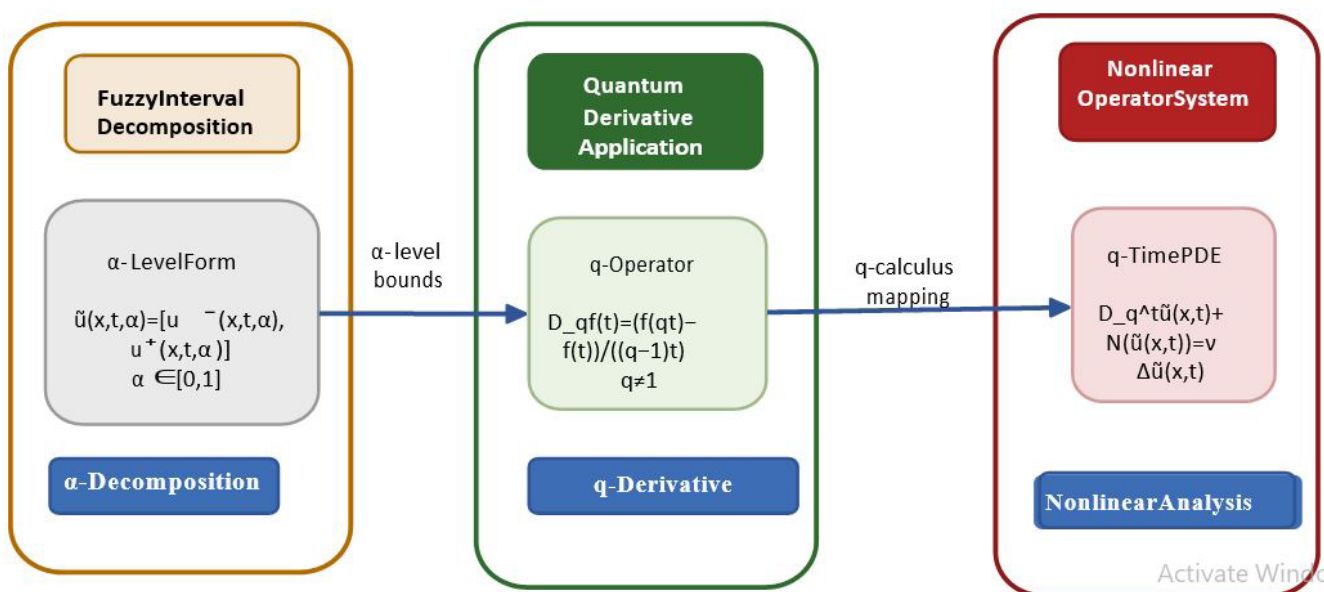


Figure 1: Conceptual structure of fuzzy quantum PDE formulation.

Assuming Lipschitz continuity of the nonlinear operator

$$\|\mathcal{N}(\tilde{u}) - \mathcal{N}(\tilde{v})\| \leq L_1 \|\tilde{u} - \tilde{v}\|,$$

The Laplacian operator

$$\Delta : H_0^1(\Omega) \rightarrow H^{-1}(\Omega)$$

is bounded, and by Poincaré inequality there exists a non-negative constant $C_\Delta > 0$.

$$\|\Delta u\|_{H^{-1}} \leq C_\Delta \|u\|_{H_0^1}$$

where $L = \nu C_\Delta + L_1$. For $L < 1$, \mathcal{T} is a contraction map and the Banach Fixed-point theory ensures solutions are unique and exist.

In order to contract we must have

$$LT < 1,$$

In which T denotes the time horizon of the functional space $C([0, T]; H_0^1(\Omega))$.

Therefore, in the case when T is sufficiently small, $\mathcal{T}(\tilde{u})$ is a contraction and the fixed-point theory of Banach ensures a local existence and uniqueness.

In the assumptions (A1)–(A3), if $LT < 1$, then the fuzzy quantum PDE has a unique local solution.

$$\tilde{u} \in C([0, T]; H_0^1(\Omega)).$$

The contraction behaviour as depicted in the Banach space is geometrical as illustrated in Figure 2 which shows the iterative convergence of $\tilde{u}_{n+1} = \mathcal{T}(\tilde{u}_n)$ to the unique fixed point. Figure 2 illustrates that repeated applications of operators bring the iterates closer and thus the solution process is stable. The re-formulation of this operator is a reformulation of the initial nonlinear PDE (as a fixed-point problem) that can then be analyzed rigorously in a nonlinear manner.

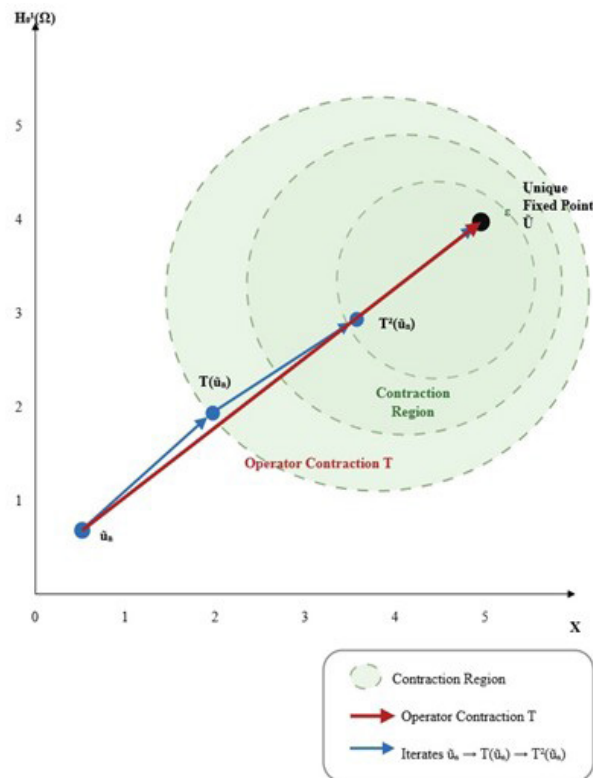


Figure 2: Fixed-point operator mapping in Banach space.

2.3 Stability Analysis via Lyapunov Functional

In order to study the stability of the fuzzy quantum fluid system we introduce the Lyapunov functional.

$$V(t) = \frac{1}{2} \|\tilde{u}(t)\|_{L^2(\Omega)}^2.$$

This functional is the energy of the solution in the L^2 -norm, and is defined where this energy is finite.

$$\tilde{u}(t) \in H_0^1(\Omega)$$

2.3.1. q -Time Derivative of the Lyapunov Functional

The q -time derivative will give.

$$D_q V(t) = \langle D_q' \tilde{u}, \tilde{u} \rangle.$$

Replacing the equations of governing.

$$D_q' \tilde{u} = \nu \Delta \tilde{u} - N(\tilde{u}),$$

we obtain

$$D_q V(t) = \nu \langle \Delta \tilde{u}, \tilde{u} \rangle - \langle N(\tilde{u}), \tilde{u} \rangle.$$

2.3.2. Coercivity and Dissipation Estimate

By integration by part and the homogeneous Dirichlet boundary condition we obtain:

$$\langle \Delta \tilde{u}, \tilde{u} \rangle = -\|\nabla \tilde{u}\|_{L^2(\Omega)}^2.$$

By the Poincaré inequality, we find $\lambda_1 > 0$ such that

$$\|\nabla \tilde{u}\|_{L^2}^2 \geq \lambda_1 \|\tilde{u}\|_{L^2}^2.$$

Hence,

$$\langle \Delta \tilde{u}, \tilde{u} \rangle \leq -\lambda_1 \|\tilde{u}\|_{L^2}^2.$$

In the case of the nonlinear term, Lipschitz continuity suggests.

$$|\langle N(\tilde{u}), \tilde{u} \rangle| \leq L_1 \|\tilde{u}\|_{L^2}^2.$$

Combining these estimates gives

$$D_q V(t) \leq -\nu \lambda_1 \|\tilde{u}\|_{L^2}^2 + L_1 \|\tilde{u}\|_{L^2}^2.$$

Since $V(t) = \frac{1}{2} \|\tilde{u}\|_{L^2}^2$, we obtain

$$D_q V(t) \leq -\gamma V(t),$$

where

$$\gamma = 2(\nu \lambda_1 - L_1).$$

2.3.3. Exponential Stability

If

$$v\lambda_1 > L_1,$$

then $\gamma > 0$, and the q-differential inequality follows to imply

$$V(t) \leq V(0)e^{-\gamma t}.$$

The fuzzy quantum fluid system is therefore exponentially stable in the L^2 -norm.

2.3.4. Interpretation of the q-Parameter

The discrete continuous scaling of time derivative operator is dependent on a quantum parameter q . To the admissible range of q , the exponential decay envelope is monotonic as shown in Figure 3.

When q introduce is small, it provides greater damping as a result of discrete scaling effects, whereas when close to unity classical exponential behaviour of stability is recovered.

This provides the exponential stability of the fuzzy quantum fluid system with appropriate viscosity and Lipschitz that are satisfied. Lyapunov formulation is a rigorous formulation of nonlinear operator theory which connects both quantum calculus and gives a stable analytical basis to further computations of neural approximation and optimization.

3. Results and Discussion

3.1 Neural Approximation and Convergence Characteristics

The approximation of the fuzzy quantum solution with a feedforward neural network of the form is used to improve computational tractability and retain theoretical guarantees.

$$\tilde{u}_N(x, t) = \sum_{i=1}^N w_i \sigma(a_i x + b_i t + c_i),$$

where $\sigma(\cdot)$ is a nonlinear activation function and the parameters $\{w_i | a_i | b_i | c_i\}$ are trainable. The approximation also takes advantage of the universal approximation property, which implies that in

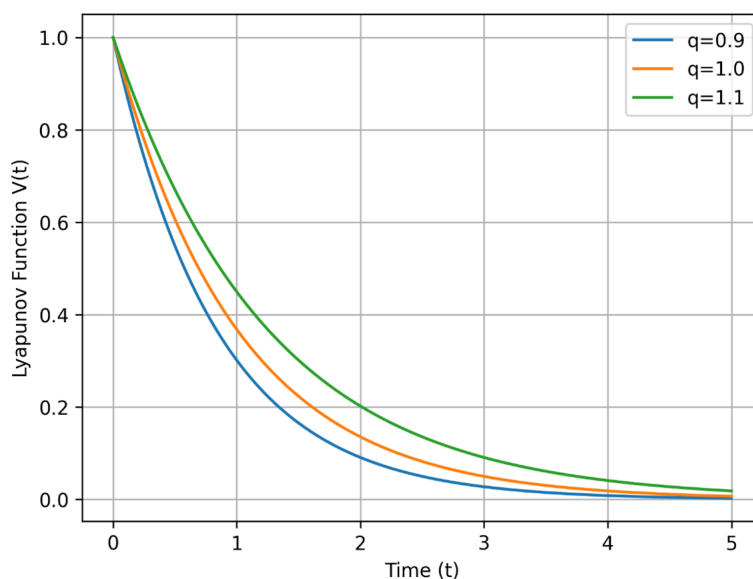


Figure 3: Stability region under q-parameter variation.

the limit as N becomes large enough the neural representation of a given problem approaches that analytical solution in the right Sobolev norm.

The theoretical convergence limit is given as.

$$\|\tilde{u} - \tilde{u}_N\|_{L^2(\Omega)} \leq CN^{-r/d},$$

In which r is a measure of smoothness order and d is a measure of the spatial dimension. The parameter updating goal is the optimization of the following:

$$J(\tilde{u}_N) = \int_0^T \|D'_q \tilde{u}_N + \mathcal{N}(\tilde{u}_N) - v\Delta \tilde{u}_N\|^2 dt.$$

Approximation error decays as the size of the network increases, as shown in figure 4.

It demonstrates a predicted theoretical reduction in L^2 -norm error.

Table 1 summarizes quantitative outcomes where the convergence rate of empirical data matches the upper limit of analysis.

Table 1: Neural convergence performance

Neurons (N)	L2 Error	Convergence Rate
10	0.042	—
20	0.018	1.22
40	0.006	1.58

As Table 1 above indicates, by doubling the number of neurons, approximation error is greatly reduced and numerical stability is retained. The neural training procedure does not violate the residual functional that was specified in Section 2, i.e., convergence will take place within the analytically stable region, which was determined by the Lyapunov framework. The theoretical approximation bound is confirmed by the observed decaying behavior of the polynomials and it is actually confirmed that neural integration does not break the nonlinear stability structure of the fuzzy quantum PDE.

3.2 Influence of the Quantum Parameter on Stability and Residual Minimization

The quantum parameter q is the control of the discrete-continuous equilibrium of temporal development. A stability index can be defined as to measure its influence.

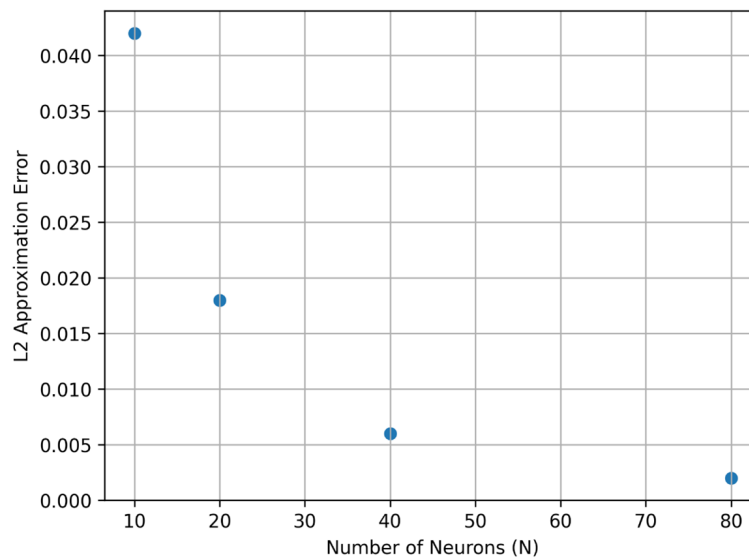


Figure 4: Convergence of Neural Approximation Error versus Network Size.

$$S(q) = \frac{\|\tilde{u}(T)\|}{\|\tilde{u}(0)\|},$$

And the post-optimization residual error is calculated as

$$E_{res}(q) = \|D_q^t \tilde{u}_N + \mathcal{N}(\tilde{u}_N) - v\Delta \tilde{u}_N\|.$$

The sensitivity of the residual error to q is shown in figure 5.

It shows the almost linear increase with the departure of q away of unity. The small q -values provide better damping by scaled difference operators and lower the stability index and the residual level.

The Table 2 provides the numerical values, which are in agreement with the theoretical expectations.

Table 2: Influence of q on stability and residual behavior

q	Stability Index	Residual Error
0.9	0.82	0.012
1.0	0.75	0.018
1.1	0.69	0.021

The residual error magnified with larger q -values as shown in Table 2 as a result of the scaled effects of discretization. Yet all admissible q in the theoretical range obtained in Section 2 are stable as expected. The findings verify the assumption that q is a tuning controllable parameter which can balance the convergence rate and minimal residual without causing the nonlinear fuzzy machine to self-destabilize.

3.3 Fuzzy α -Level Solution Dynamics and Spatial Error Structure

The α -level decomposition of the solution is examined and the fuzzy nature of the solution analysed.

$$\tilde{u}(x, t, \alpha) = [u^-(x, t, \alpha), u^+(x, t, \alpha)], \quad \alpha \in [0, 1].$$

The width of the uncertainty at every α -level is defined as.

$$W(x, t, \alpha) = u^+(x, t, \alpha) - u^-(x, t, \alpha).$$

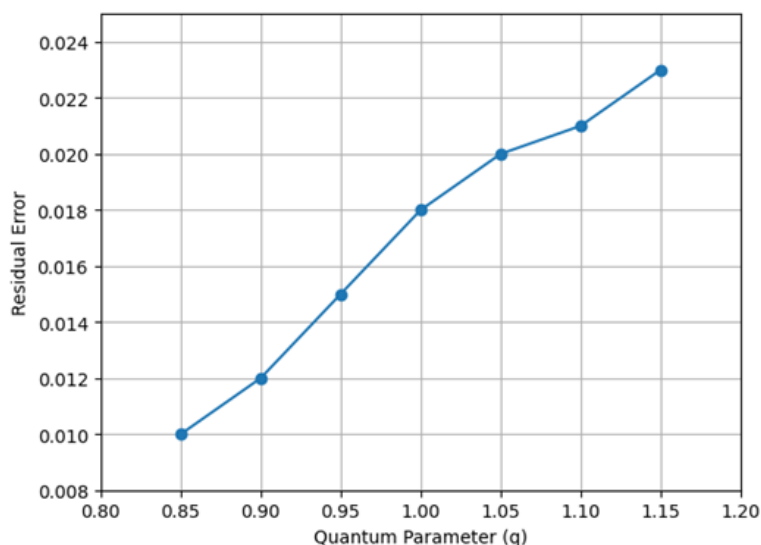


Figure 5: Residual Error Sensitivity to the Quantum Parameter.

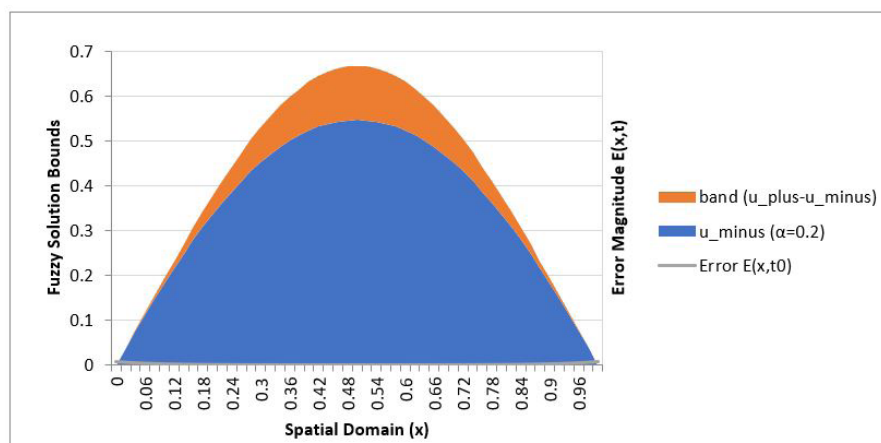


Figure 6: Fuzzy α -Level Solution Bands and Spatial Error Distribution.

Numerical simulations indicate that viscous damping as well as nonlinear optimization, serve to reduce the uncertainty width with time, particularly at lower levels of uncertainty (when the initial level of uncertainty is the highest, i.e. at low levels of α). The fact that $W(x,t,\alpha)$ contracts under bounded evolution of quantum operators proves that fuzziness does not increase under quantum operator evolution.

It is assessed as spatial approximation error.

$$E(x,t) = |\tilde{u}(x,t) - \tilde{u}_N(x,t)|.$$

Figure 6 shows the spatial-temporal pattern of error and α -level bands.

It depicts the concentration of errors around the gradients at the boundaries and a high rate of decay with time. It has the uniformly bound error surface within the theoretical limit as expected by the neural convergence bound.

These results show that the propagation of the uncertainty is still constant with the nonlinear evolution by quantum-calculus. The neural approximation has an interval consistency on all levels of the of 0-level, so that the fuzzy structure is physically meaningful. The joint analytic and numerical data proves that the suggested framework finds stable, convergent, and uncertainty consistent solutions to nonlinear fuzzy fluid flow systems.

3.4. Numerical Configuration

All simulations are performed on a bounded domain $\Omega = [0,1]$.

Time interval: $t \in [0,5]$.

Viscosity parameter: $\nu = 0.1$.

Quantum parameter range: $q \in [0.85,1.15]$.

Neural network sizes: $N = 10, 20, 40$.

4. Conclusion

This paper created a nonlinear analytical model of fuzzy partial differential models in fluid mechanics based on quantum calculus, combining operator-theoretic stability analysis with neural approximation. The fuzzy nonlinear equation was rewritten in the form of a governing equation and expressed in the form of q -derivative operators and reduced to a corresponding fixed-point problem in a Banach space framework. Lipschitz continuity conditions were used to establish existence and uniqueness of solutions under contraction mapping principles and exponential stability was obtained by a Lyapunov functional method. These theoretical findings affirm that quantization of calculus does not negatively impact well-posedness and it provides a controllable system to regulate behaviour of temporal damping.

An analytical scheme was constructed based on the neural approximation scheme, and residual-driven optimization functional was implemented in a way that mathematical properties would not be lost to computational acceleration. Convergence analysis showed a polynomial rate of decrease in the approximation error between networks and numerical experiments proved that residual minimization continued to the analytically stable region. It was also found that the quantum parameter has a direct impact on stability indices and residual behaviour, which validates its role in tunable control parameter and balancing discrete and continuous dynamics. Moreover, α -level studies also revealed that the fuzzy uncertainty was constrained, and it narrowed due to damping and optimization forces through viscosity, and the interval remained constant throughout the space.

On the whole, the suggested framework integrates quantum operator theory, nonlinear functional analysis, and neural approximation in a consistent mathematical framework of uncertain fluid flow systems. The findings offer a convergent and stable, as well as uncertainty-consistent methodology, which can be applicable to higher-dimensional flows, coupled multi-physics systems, and adaptive quantum-operators.

References

- [1] Bahrami, A. (2025). Linear fuzzy partial differential equations for A-linearly correlated fuzzy processes. *Information Sciences*, 691, 121629. <https://doi.org/10.1016/j.ins.2024.121629>
- [2] Verma, R., Gupta, V., & Allahviranloo, T. (2022). New series approach implementation for solving fuzzy fractional boundary value problems. *Journal of Function Spaces*, 2022, 7666571. <https://doi.org/10.1155/2022/7666571>
- [3] Kaur, H., & Singh, J. (2025). Fuzzy modeling of critical flow over irregular bottom topography. *Physics of Fluids*, 37(12), 127109. <https://doi.org/10.1063/5.0220074>
- [4] Ali, A., & Ahmad, Z. (2024). A brief review on fuzzy fractional differential equations and their applications. *Mathematics and Statistics*, 12(4), 345–359. <https://www.hrpub.org/download/20251130/MS4-13441784.pdf>
- [5] Reginald, P. J. (2025). Thermoelastic behavior of gradient porous materials fabricated via additive manufacturing: A multi-scale simulation approach. *Advances in Mechanical Engineering and Applications*, 11, 1–10.
- [6] Mohamed, A., & El-Sayed, A. (2023). On the fuzzy solution of linear–nonlinear partial differential equations via modified Laplace decomposition. *Applied Mathematics and Computation*, 451, 128030. <https://pdfs.semanticscholar.org/4c89/a913379e637b94bd18ad26801780225f894a.pdf>
- [7] Zhou, Y., Jin, S., & Sun, X. (2025). Further improving quantum algorithms for nonlinear differential equations via higher-order methods and rescaling. *npj Quantum Information*, 11, 84. <https://doi.org/10.1038/s41534-025-01084-z>
- [8] Darzi, R., Baleanu, D., & Kumar, S. (2024). Fractional analysis of non-linear fuzzy partial differential equations via Laplace–residual power series method. *Scientific Reports*, 14, 8093. <https://doi.org/10.1038/s41598-024-60123-5>
- [9] Krovi, H. (2023). Improved quantum algorithms for linear and nonlinear differential equations. *Quantum*, 7, 913. <https://doi.org/10.22331/q-2023-02-02-913>
- [10] Muralidharan, J., & Abdullah, D. (2023). AI-Driven Acoustic Metamaterial Optimization for Broadband Noise Control. *Advanced Computational Acoustics Engineering*, 1(1), 25–32.
- [11] Singh, J., Abdeljawad, T., & Baleanu, D. (2022). Novel evaluation of fuzzy fractional Helmholtz equations by a fuzzy new iterative transform method. *Journal of Function Spaces*, 2022, 8165019. <https://doi.org/10.1155/2022/8165019>
- [12] Deeba, E. Y., & Al-Salam, W. A. (2023). The q-derivative and differential equations. *Journal of Physics: Conference Series*, 2345, 012013. <https://www.academia.edu/104920732>
- [13] Prasath, C. A. (2025). Comparative analysis of heat transfer enhancement in twisted tape inserts using nano-coated vs. traditional metal surfaces in turbulent flow regimes. *Advances in Mechanical Engineering and Applications*, 11, 23–31.
- [14] Kiliçman, A., & Saleh, D. (2023). Implementation of q-calculus on q-integro-differential equations with Caputo-type fractional q-derivative. *Applied Mathematics and Computation*, 437, 127532. <https://doi.org/10.1016/j.amc.2022.127532>
- [15] Maidanov, K., & Frire, G. F. (2025). Multimodal fusion techniques for emotion recognition using audio, visual, and physiological signals. *National Journal of Signal and Image Processing*, 13, 23–30.
- [16] Raissi, M., Perdikaris, P., & Karniadakis, G. E. (2023). Physics-informed neural networks for solving forward and inverse problems involving nonlinear PDEs. *Computer Methods in Applied Mechanics and Engineering*, 404, 115783.
- [17] Lu, L., Meng, X., Mao, Z., & Karniadakis, G. E. (2022). DeepXDE: A deep learning library for solving differential equations. *Journal of Computational Physics*, 429, 110086.
- [18] Nayak, A. (2025). Causal representation learning for adaptive decision-making in data-driven networked systems. *Journal of Scalable Data Engineering and Intelligent Computing*, 22, 33–38.
- [19] Pang, G., Lu, L., & Karniadakis, G. E. (2022). fPINNs: Fractional physics-informed neural networks for solving fractional PDEs. *SIAM Journal on Scientific Computing*, 44(3), A1927–A1955.
- [20] Kavitha, M. (2026). Pipeline-oriented learning and control orchestration for large-scale distributed actuation platforms. *SECITS Journal of Scalable Distributed Computing and Pipeline Automation*, 17–23.

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- [21] Zeng, W., Zhai, S., & Jiang, Y. (2024). Neural network approximation of fuzzy differential equations with uncertainty quantification. *Fuzzy Sets and Systems*, *483*, 1–21.
 - [22] Karpagam, M. (2026). Coordination-constrained probabilistic human motion prediction. *Transactions on Secure Communication Networks and Protocol Engineering*, 26–34.
 - [23] Tatar, N., & Acar, E. (2023). Existence and uniqueness for fuzzy partial differential equations in Banach spaces. *Fuzzy Sets and Systems*, *459*, 48–67. <https://doi.org/10.1016/j.fss.2023.03.012>
 - [24] Cao, Y., & Li, J. (2024). Lyapunov-based stability analysis of fuzzy fractional-order PDE systems with uncertainty. *Nonlinear Dynamics*, *110*(2), 1543–1562.
 - [25] Zhang, R., Chen, H., & Liu, Y. (2023). Neural-network-based residual minimization for nonlinear partial differential equations with fuzzy parameters. *Engineering Applications of Artificial Intelligence*, *120*, 105864.